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License and Entry Decisions for a Firm with a Cost Advantage in an International Duopoly under Convex Cost Functions

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We consider the following options for a foreign firm with a cost advantage: licensing its cost-reducing technology to a domestic incumbent firm under a fixed fee, or entering the domestic market with or without license under convex cost functions. Under such functions, the domestic and foreign markets are not separated, and the outcomes depend on the relative size of these markets. In a specific case with a linear demand function and a quadratic cost function, entry without a license is not the optimal strategy for the foreign firm with the cost advantage. If the ratio of the size of the foreign market to that of the domestic market is small, license with entry may be the optimal strategy. In contrast, if the ratio of the size of the foreign market to that of the domestic market is not small, or the cost advantage is significant, license without entry may be the optimal strategy.

Keywords: license with entry, license without entry, duopoly, foreign market, domestic market, cost advantage

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1 Introduction

Proposition 4 in Kamien and Tauman (1986) assumes linear demand and cost functions, a fixed license fee, and cost-reducing technology in an oligopoly. License with entry is a strategy of a firm with a cost advantage (a licensor) to enter the market and simultaneously license its cost-reducing technology to an incumbent firm (a licensee). License without entry is defined as a strategy for the licensor to license its technology to the licensee without entering the market. Given these definitions, Proposition 4 states that in an oligopoly, when the number of firms is small (or large), license with entry by a licensor is more profitable than license without entry. We believe that their definition of license fee in the case where a licensor licenses its technology to a licensee and does not enter the market is not appropriate. Their analysis under a duopoly model defines the license fee as the difference between the licensee's profit and its monopoly profit before entry and licensing. However, we believe that if the negotiation of the license fee between the licensor and the licensee breaks down, the licensor can enter the market without licensing the licensee. If the licensor does not enter the market or license the licensee, its profit is zero. However, if it enters the market, its profit becomes positive. As this threat of entry is credible, the licensee must pay a license fee equal to the difference between its profit in the cases of license without entry and entry without license.

We consider options for a foreign firm with a cost advantage in licensing its cost-reducing technology to a domestic incumbent firm or entering the domestic market with or without licensing the domestic firm, under convex cost functions using the definition of license fee with respect to the above mentioned point.

Convex cost functions represent increasing marginal costs. When the efficient inputs are exhausted, managers have to use inefficient inputs to produce more output. That is, producing another unit of output incurs a higher cost than before and as such, the marginal cost increases. Such a situation might occur in a restaurant located in a small town with a few good cooks or in an electric company with only a few productive power plants.

Another reason for an increase in marginal cost is as follows. To increase output from a given production process, firms may have to incur expenses in terms of overtime charges paid to workers or input costs, or use a more expensive production system.

With convex cost functions, the domestic and foreign markets are not separated, and the results depend on the relative size of these markets. In a specific case with a linear demand function and a quadratic cost function, entry without license is not the optimal strategy for the firm with a cost advantage. If the ratio of the size of the foreign market to that of the domestic market is small, license with entry may be the optimal strategy. In contrast, if the ratio of the size of the foreign market to that of the domestic market is not small or the cost advantage is significant, license without entry is the optimal strategy.

In the next section, we briefly review some related studies. In Section 3, we present the model. In Section 4, we study the general case, and in Section 5, we investigate the optimal strategies for the foreign firm with a cost advantage (the licensor) in the case of a linear demand function and a quadratic cost function. In Section 6, we examine the effects of a strict capacity limit and transportation cost on the optimal strategy of the licensor.

2 Literature Review

We present a brief review of studies that analyze related topics. La Manna (1993) analyzed a Cournot oligopoly with fixed fee under cost asymmetry and demonstrated that if technologies can be replicated perfectly, a lower cost firm always has the incentive to transfer its technology. Hence, while a Cournot-Nash equilibrium cannot be fully asymmetric, there is no non-cooperative Nash equilibrium in pure strategies. On the other hand, using cooperative game theory, Watanabe and Muto (2008) analyzed bargaining between a licensor with no production capacity and oligopolistic firms. Recent research focuses on market structure and technology improvement. Boone (2001) and Matsumura *et al*. (2013) found a non-monotonic relation between the intensity of competition and innovation. Further, Pal (2010) demonstrated that technology adoption might change the market outcome. Social welfare is larger in Bertrand competition than in Cournot competition. However, if we consider technology adoption, Cournot competition may result in higher social welfare than Bertrand competition in a market for differentiated goods. Hattori and Tanaka (2014, 2016a) studied the adoption of cost-reducing technology in Cournot and Stackelberg duopolies. Rebolledo and Sandon (2012) analyzed the effectiveness of research and development (R&D) subsidies in an oligopolistic model in the cases of international competition and cooperation in R&D. Hattori and Tanaka (2016b) analyzed similar problems in product innovation, that is, introduction of higher quality goods in a duopoly with vertical product differentiation. Hattori and Tanaka (2017) examine the definition of a license fee when the innovator can choose whether to enter the market in a duopoly with vertically differentiated goods.

We believe that licensing and entry strategies by an outside licensor with the possibility of entering the market are not analyzed in any research other than above mentioned studies. We extend this analysis to an international framework.

3 The Model

There are two countries and two firms, Firm A in Country A, which is the foreign firm, and Firm B in Country B, which is the domestic firm. Currently, each firm produces the same good in their respective countries. However, Firm A has a superior cost-reducing technology and can produce the good at lower cost than Firm B.

Firm A has three options: 1) enter the domestic market without licensing its technology to Firm B; 2) license its technology to Firm B without entering the domestic market; and 3) enter the domestic market and license its technology to Firm B. If Firm A enters the domestic market, it becomes a duopoly. As the focus of this paper is a choice of entry or license by Firm A, we assume that Firm B does not enter the foreign market. Let p be the price and X be the total supply in the domestic market. The inverse demand function is written as

$$p = p(X).$$

The supplies of Firm A and Firm B are denoted by x_A and x_B, respectively. Thus, $X = x_A + x_B$. In the foreign market, the supply of Firm A and the price of the good are denoted by y_A and q, respectively. The inverse demand function is written as

$$q = q\left(\frac{y_A}{t}\right),$$

where t is a positive number that represents the ratio of the size of the foreign market to that of the domestic market. If $t < 1$, the size of the foreign market is smaller than the size of the domestic market. If $t > 1$, the size of the foreign market is larger than the size of the domestic market.

We assume that the cost functions of Firm A and Firm B are convex. They are $c_A(x_A + y_A)$ and $c_B(x_B)$ before licensing. After Firm B gets the license, its cost function is the same as that of Firm A, that is $c_A(x_B)$.

Structure of the Game

The structure of the game is as follows.

(1) In the first stage, Firm A offers a license to use its cost-reducing technology to Firm B at some license fee on the condition that Firm A either enters or does not enter the domestic market on acceptance of the offer. This decision of entry by Firm A should be credible.

(2) In the second stage, Firm B decides whether to accept the offer by Firm A. If it refuses the offer, Firm A decides whether to enter the domestic market without license.

(3) In the third stage, both Firm A and Firm B simultaneously determine their supplies to each market.

The license fee is determined in the first stage based on Firm B's willingness to pay, which is equal to its profit (before paying the license fee) when it accepts the offer, and its profit when it refuses the offer under the threat of entry by Firm A. Firm A determines its supply to both the markets in the third stage of the game given the licensing decision, while in the first stage it decides whether to enter the domestic market when it sells a license.

4 General Analysis

4.1 Firms' Behavior

1. When Firm A enters the domestic market without licensing to Firm B, the profits

of Firm A and Firm B are denoted as

$$\pi_A^e = px_A + qy_A - c_A(x_A + y_A),$$

and

$$\pi_B^e = px_B - c_B(x_B).$$

The conditions for profit maximization of Firm A and Firm B are

$$p + x_A p' - c_A'(x_A + y_A) = 0,$$
$$q + \frac{y_A}{t} q' - c_A'(x_A + y_A) = 0,$$

and

$$p + x_B p' - c_B'(x_B) = 0.$$

2. When Firm A licenses its technology to Firm B without entering the domestic market, the profits of Firm A and Firm B in the market are denoted by

$$\pi_A^l = qy_A - c_A(y_A),$$

and

$$\pi_B^l = px_B - c_A(x_B) - L^l,$$

where L^l is the license fee, π_B^l denotes Firm B's profit after paying the license fee, and $\pi_B^l + L^l$ is Firm B's profit before paying the license fee. As Firm A and Firm B use the cost reducing technology, we denote the cost functions of both the firms by c_A. The conditions for profit maximization of Firm A and Firm B are

$$q + \frac{y_A}{t} q' - c_A'(y_A) = 0,$$

and

$$p + x_B p' - c_A'(x_B) = 0.$$

If the negotiation of the license fee between Firm A and Firm B breaks down, Firm A can enter the domestic market without licensing Firm B. Therefore, Firm B must pay the difference between its profit, excluding the license fee, and its profit in the case of entry without license, such that it satisfies the following condition.

$$L^{l}=\pi_{B}^{l}+L^{l}-\pi_{B}^{e}. \quad (1)$$

This equation implies that the license fee is determined such that $\pi_B^l = \pi_B^e$ holds. Thus, Firm A’s total profit is given by

$$\pi_A^l + L^l.$$

3. When Firm A enters the domestic market and at the same time licenses its technology to Firm B, the profits of Firm A and Firm B are

$$\pi_{A}^{el}=px_{A}+qy_{A}-c_{A}(x_{A}+y_{A}),$$

and

$$\pi_A^{el} = px_B - c_A(x_B) - L^{el}.$$

Both Firm B and Firm A use the cost reducing technology. Thus, we denote the cost functions of both firms by c_A in this case as well. L^{el} is the license fee and π_A^{el} is Firm B’s profit after paying the license fee. Before paying the license fee, it is $\pi_A^{el} + L^{el}$. Thus, the conditions for profit maximization of Firm A and Firm B are

$$p+x_{A}p'-c_{A}'(x_{A}+y_{A})=0,$$
$$q+\frac{y_{A}}{t}q'-c_{A}'(x_{A}+y_{A})=0,$$

and

$$p + x_B p' - c_A{}'(x_B) = 0.$$

Similarly, if the negotiation of the license fee between Firm A and Firm B breaks down, Firm A can enter the domestic market without a license. Therefore, Firm B must pay the difference between its profit, excluding the license fee, and its profit in the case of entry without license, such that it satisfies the following condition.

$$L^{el} = \pi_B^{el} + L^{el} - \pi_B^e. \quad (2)$$

This equation implies that the license fee is determined such that $\pi_B^{el} = \pi_B^e$ holds. Thus, Firm A’s total profit is given by

$$\pi_A^{el} + L^{el}.$$

4.2 The Optimal Strategies

Comparing $\pi_A^l + L^l$, π_A^e and $\pi_A^{el} + L^{el}$, the optimal strategies for Firm A are as follows.

1. If $\pi_A^l + L^l$ is the maximum, license without entry strategy is optimal.
2. If π_A^e is the maximum, entry without license strategy is optimal.
3. If $\pi_A^{el} + L^{el}$ is the maximum, license with entry strategy is optimal.

5 Linear Demand Function and Quadratic Cost Function

5.1 Demand and Cost Functions

We consider a case with a linear demand function and a quadratic cost function. The inverse demand function in the domestic market is

$$p=a-X,$$

where a is a positive constant. The inverse demand function in the foreign market is

$$q = a - \frac{y_A}{t}.$$

The cost functions of Firm A and Firm B before the license are $c_A(x_A + y_A)^2$ and $c_B x_B^2$, where c_A and c_B are positive constants such that $c_A < c_B$. After the license, the cost function of Firm B is $c_A x_B^2$.

5.2 Equilibrium Outputs and Profits

The equilibrium outputs and profits are as follows.

1. Entry without license

Suppose Firm A enters the domestic market without licensing Firm B. The profits of Firm A and Firm B are, respectively

$$\pi_A = p x_A + q y_A - c_A(x_A + y_A)^2,$$

and

$$\pi_B = px_B - c_B x_B^2.$$

The equilibrium outputs and profits are obtained as follows.

$$x_A = \frac{a(2c_B + 1 - c_A t)}{4c_A c_B t + 3c_A t + 4c_A c_B + 4c_B + 4c_A + 3},$$

$$y_A = \frac{a(4c_B + 2c_A + 3)t}{2(4c_A c_B t + 3c_A t + 4c_A c_B + 4c_B + 4c_A + 3)},$$

$$x_B = \frac{a(2c_A t + 2c_A + 1)}{4c_A c_B t + 3c_A t + 4c_A c_B + 4c_B + 4c_A + 3},$$

$$\pi_A^e = \frac{a^2 \lambda_A}{4(4c_A c_B t + 3c_A t + 4c_A c_B + 4c_B + 4c_A + 3)^2},$$

$$\pi_B^e = \frac{a^2 (c_B + 1)(2c_A t + 2c_A + 1)^2}{(4c_A c_B t + 3c_A t + 4c_A c_B + 4c_B + 4c_A + 3)^2}.$$

All λ's are in Appendix.

If $t > (2c_B + 1)/c_A$, then $x_A = 0$. Thus, Firm A does not enter the domestic market even without licensing Firm B.

2. License without entry

Suppose Firm A licenses its technology to Firm B without entering the domestic market. The profits of the Firm A and Firm B are, respectively

$$\pi_A = qy_A - c_A y_A^2,$$

and

$$\pi_B = px_B - c_A x_B^2 - L^l.$$

The equilibrium outputs and profits are

$$y_A = \frac{at}{2(c_A t + 1)},\ x_B = \frac{a}{2(c_A + 1)},\ \pi_A^l = \frac{a^2 t}{4(c_A t + 1)},\ \pi_B^l = \frac{a^2}{4(c_A + 1)} - L^l.$$

The license fee is equal to

$$L^l = \frac{a^2 \lambda_B}{4(c_A + 1)(4c_A c_B t + 3c_A t + 4c_A c_B + 4c_B + 4c_A + 3)^2}.$$

Firm A’s total profit including the license fee is

$$\pi_A^l + L^l = \frac{a^2\lambda_C}{4(c_A+1)(c_At+1)(4c_Ac_Bt+3c_At+4c_Ac_B+4c_B+4c_A+3)^2}.$$

3. Entry with license

Suppose Firm A enters the domestic market and simultaneously licenses its technology to Firm B. The profits of Firm A and Firm B are, respectively

$$\pi_A = px_A + qy_A - c_A(x_A + y_A)^2,$$

and

$$\pi_B = px_B - c_Ax_B^2 - L^{el}.$$

The equilibrium outputs and profits are

$$x_A = \frac{a(2c_A+1-c_At)}{4c_A^2t+3c_At+4c_A^2+8c_A+3},$$

$$x_B = \frac{a(2c_At+2c_A+1)}{4c_A^2t+3c_At+4c_A^2+8c_A+3},$$

$$y_A = \frac{3a(2c_A+1)t}{2(4c_A^2t+3c_At+4c_A^2+8c_A+3)},$$

$$\pi_A^{el} = \frac{a^2\lambda_D}{4(4c_A^2t+3c_At+4c_A^2+8c_A+3)^2},$$

$$\pi_B^{el} = \frac{a^2(c_A+1)(2c_At+2c_A+1)^2}{(4c_A^2t+3c_At+4c_A^2+8c_A+3)^2} - L^{el}.$$

The license fee is equal to

$$L^{el} = \frac{a^2(c_B-c_A)(2c_At+2c_A+1)^2\lambda_E}{(4c_A^2t+3c_At+4c_A^2+8c_A+3)^2(4c_Ac_Bt+3c_At+4c_Ac_B+4c_B+4c_A+3)^2}.$$

Firm A’s total profit including the license fee is

$$\pi_A^{el} + L^{el} = \frac{a^2\lambda_F}{4(4c_A^2t+3c_At+4c_A^2+8c_A+3)^2(4c_Ac_Bt+3c_At+4c_Ac_B+4c_B+4c_A+3)^2}.$$

If $t > (2c_A + 1)/c_A$, then $x_A = 0$. Thus, Firm A does not enter the domestic market when it sells a license. We can observe that $(2c_B + 1)/c_A > (2c_A + 1)/c_A$.

5.3 The Optimal Strategies

As regards the value of t, there are three cases.

Case 1. If $t > (2c_B + 1)/c_A$, then, Firm A never enters the domestic market, and the cases of entry with and without license do not exist.

Case 2. If $(2c_A + 1)/c_A < t \le (2c_B + 1)/c_A$, then, Firm A does not enter the domestic market with license to Firm B, and the case of entry with license does not exist.

Case 3. If $t \le (2c_A + 1)/c_A$, then, Firm A may enter the domestic market with or without licensing.

We consider the optimal strategies for Firm A in each case.

Case 1. If $t > (2c_B + 1)/c_A$, then Firm A never enters the domestic market. Thus, its optimal strategy is license without entry. The license fee is equal to the difference between Firm B's profit in this case and its profit before licensing or entry by Firm A, that is, Firm B's monopoly profit with the high cost technology.

Case 2. If $(2c_A + 1)/c_A < t \le (2c_B + 1)/c_A$, then Firm A does not enter the domestic market when it licenses its technology to Firm B. Comparing Firm A's profit in this case and its profit when it enters the market without licensing, we get

$$\pi_A^l + L^l - \pi_A^e = \frac{\lambda_G}{4(c_A + 1)(c_A t + 1)(4c_A c_B t + 3c_A t + 4c_A c_B + 4c_B + 4c_A + 3)^2}. \quad (3)$$

This is positive for reasonable values of variables if $t > (2c_A + 1)/2$. Firm A's total profit when it licenses its technology to Firm B without entry is larger than its total profit when it enters the domestic market without licensing Firm B, and license without entry is the optimal strategy. In Figure 1, we depict examples of this case assuming $c_B = 5$ for $c_A = 1, c_A = 2$, and $c_A = 4$. When $c_A = 1, (2c_A + 1)/c_A$=3; when $c_A = 2, (2c_A + 1)/c_A$ =2.5; and when $c_A = 4, (2c_A + 1)/c_A$ =2.25. When $c_A = 1, (2c_B + 1)/c_A$ =11; when $c_A = 2, (2c_B + 1)/c_A$ =5.5; and when $c_A = 4, (2c_B + 1)/c_A$=2.75. Further, in Figure 2, we present other examples assuming

$c_B = 10$ for $c_A = 1, c_A = 4$ and $c_A = 8$. When $c_A = 8, (2c_A + 1)/c_A$=2.1. When $c_A = 1, (2c_B + 1)/c_A$ =21, when $c_A = 4$, $(2c_B + 1)/c_A$ =5.25, and when $c_A = 8, (2c_B + 1)/c_A$=2.625. The thick line, the medium line, and the thin line in Figure 1 represent the value of $\pi_A^l + L^l - \pi_A^e$ when $c_A = 4$, 1 and 2. Similarly, the thick line, the medium line, and the thin line in Figure 2 represent the value of $\pi_A^l + L^l - \pi_A^e$ when $c_A = 8$, 1, and 4.

Discussion on Case 2

The difference between Firm A's profit in the case of license without entry and that in the case of entry without license is

$$\pi_A^l + L^l - \pi_A^e = \pi_A^l - \pi_A^e + \pi_B^l - \pi_B^e .$$

When Firm A licenses its technology to Firm B, the domestic market becomes a monopoly in which Firm B produces the good at a lower cost. Then, π_B^l is larger than the sum of π_B^e and Firm A's profit in the domestic market when it enters the domestic market. It implies that L^l is larger than Firm A's profit in the domestic market. Further, Firm A's supply in the foreign market in the case of license without entry is larger than that in the case of entry without license under convex cost functions. Let y_A^l and y_A^e be the supplies in the foreign market in the cases of license without entry and entry without license, respectively.

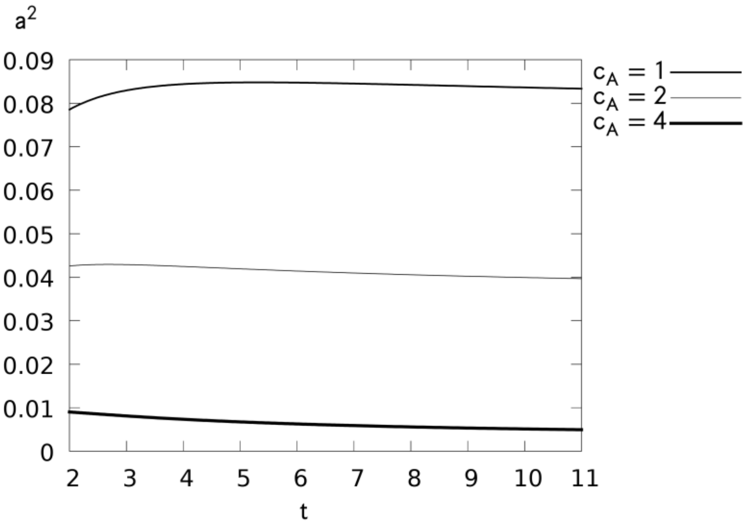

Figure 1. Illustration of $\pi_A^l + L^l - \pi_A^e$: $c_B = 5$

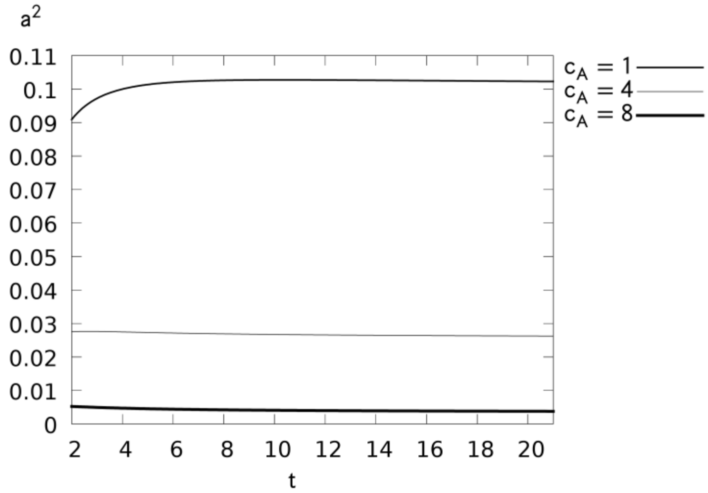

Figure 2. Illustration of $\pi_A^l + L^l - \pi_A^e$: $c_B = 10$

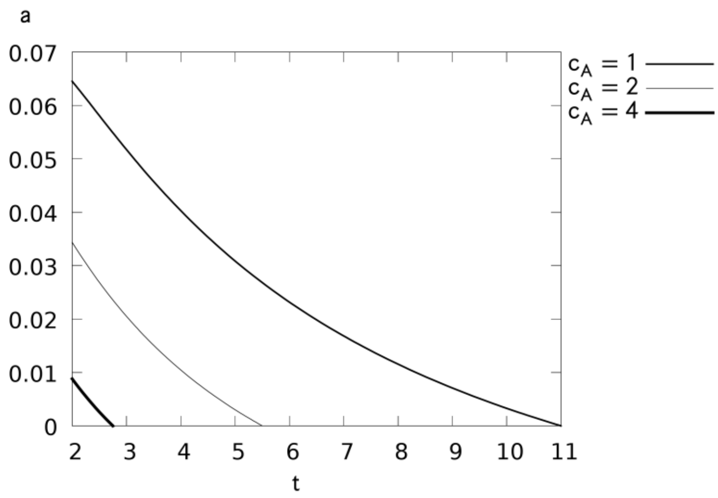

Figure 3. Illustration of $y_A^l - y_A^e$

See Figure 3 for an illustration of $y_A^l - y_A^e$ when $c_B = 5$, for $c_A = 1$, $c_A = 2$, and $c_A = 4$. When $c_A = 1$, we consider a case with $3 \leq t \leq 11$; when $c_A = 2$, we consider a case with $2.75 \leq t \leq 5.5$; and when $c_A = 4$, we consider a case with $2.25 \leq t \leq 2.75$.

Case 3. Now, consider the case where $t \leq (2c_A + 1)/c_A$. Let us compare Firm A's profit when it enters the domestic market with and without license to Firm B. We have

$$\pi_A^{el} + L^{el} - \pi_A^e = \frac{(c_B - c_A)(2c_A t + 2c_A + 1)\lambda_H}{(4c_A^2 t + 3c_A t + 4c_A^2 + 8c_A + 3)^2 (4c_A c_B t + 3c_A t + 4c_A c_B + 4c_B + 4c_A + 3)^2}. \quad (4)$$

This is positive. Thus, entry only (entry without license) strategy is never the optimal strategy for Firm A. In Figure 4, we depict examples of $\pi_A^{el} + L^{el} - \pi_A^e$ assuming $c_B = 5$ for $c_A = 1$, $c_A = 2$ and $c_A = 4$.

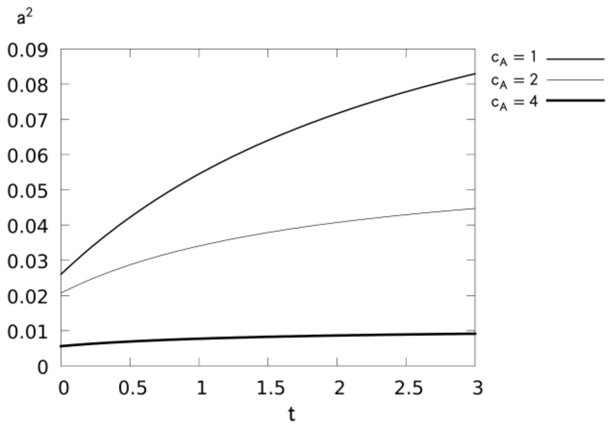

Figure 4. Illustration of $\pi_A^{el} + L^{el} - \pi_A^e$

Comparing Firm A's profit when it enters the domestic market with license to Firm B and its profit when it licenses its technology to Firm B without entry yields

$$\pi_A^{el} + L^{el} - (\pi_A^l + L^l) = \frac{a^2(c_A t - 2c_A - 1)\lambda_I}{4(c_A+1)(c_A t+1)(4c_A^2 t + 3c_A t + 4c_A^2 + 8c_A + 3)^2}. \quad (5)$$

This difference depends on the values of t and c_A, but does not depend on the value of c_B. Solving $\pi_A^{el} + L^{el} - (\pi_A^l + L^l) = 0$, we obtain the following solution.

$$t^* = \frac{(c_A+1)\sqrt{100c_A^2 + 68c_A + 25} - 2c_A^2 - 9c_A - 6}{12c_A^2 + 11c_A}.$$

This is the threshold value of the relative size of the foreign market to that of the domestic market. It only depends on c_A. If $t < t^*$, then $\pi_A^l + L^l < \pi_A^{el} + L^{el}$, and if $t > t^*$, then $\pi_A^l + L^l > \pi_A^{el} + L^{el}$. Thus, if the foreign market is small relative to the domestic market, license with entry is the optimal strategy for Firm A, and if the foreign market is not small relative to the domestic market, license without entry is the optimal strategy. In Figure 5, we depict examples of $\pi_A^{el} + L^{el} - (\pi_A^l + L^l)$ for $c_A = 1,\ 2,$ and 4. When $c_A = 1$, we consider a case where $0 < t < 3$; when $c_A = 2$, we consider a case where $0 < t < 2.5$; and when $c_A = 4$, we consider a case where $0 < t < 2.25$. We have $t^* = 0.469$ for $c_A = 1$, $t^* = 0.558$ for $c_A = 2,$ and $t^* = 0.609$ for $c_A = 4$.

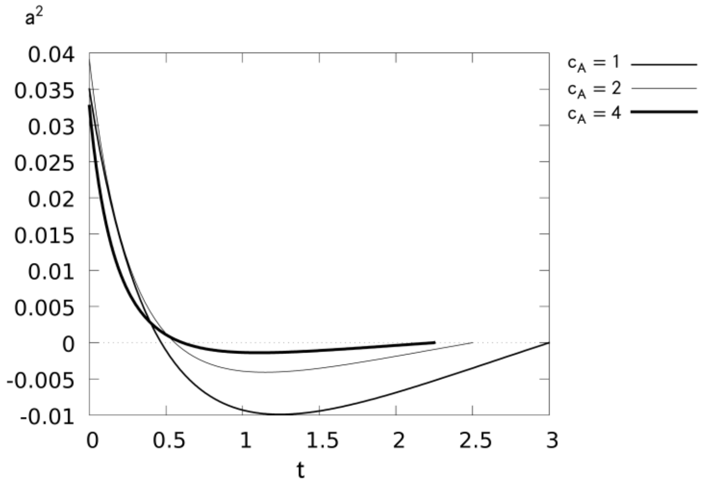

Figure 5. Illustration of $\pi_A^{el} + L^{el} - (\pi_A^l + L^l)$

When $c_A \le (\sqrt{2} - 1)/2$, $t^* \le 0$. Therefore, if $c_A \le (\sqrt{2} - 1)/2$, there is no positive t^* and license without entry is the optimal strategy. In Figure 6, we depict the relationship between c_A and t^*. As $c_A \to \infty$, $t^* \to 2/3$.

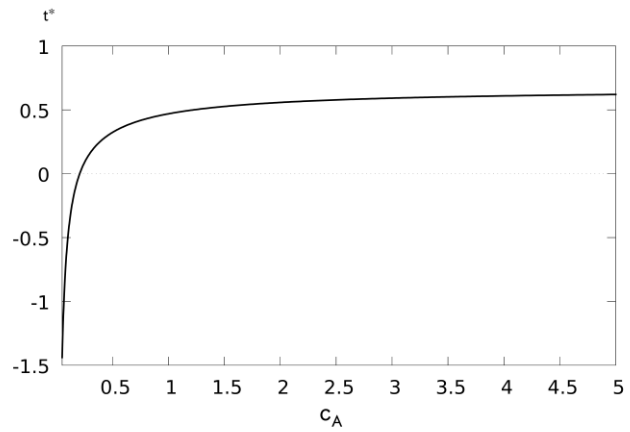

Figure 6. Relationship between c_A and t^*

Discussion on Case 3

Since $L^{el} - L^l = \pi_B^{el} + L^{el} - (\pi_B^l + L^l)$, (5) is rewritten as

$$\pi_A^{el} + L^{el} - (\pi_A^l + L^l) = \pi_A^{el} - \pi_A^l + \pi_B^{el} + L^{el} - (\pi_B^l + L^l).$$

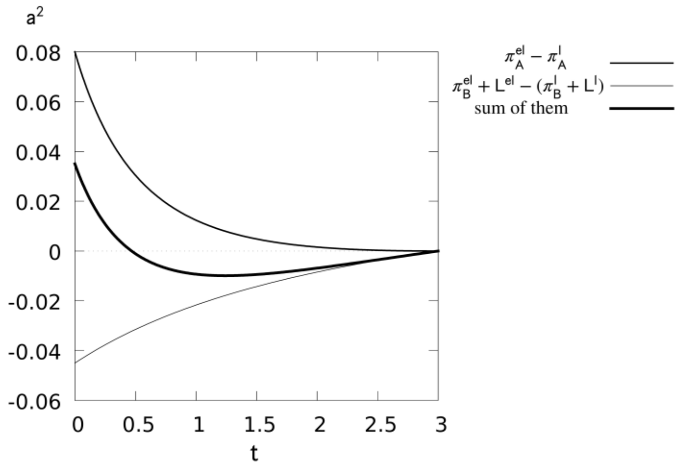

Figure 7. Relationship among $\pi_A^{el} - \pi_A^l$, $\pi_B^{el} + L^{el} - (\pi_B^l + L^l)$ and t

In Figure 7, we illustrate the relationships among $\pi_A^{el} - \pi_A^l$, $\pi_B^{el} + L^{el} - (\pi_B^l + L^l)$, and t, assuming that $c_A = 1$ and $c_B = 5$. It indicates that $\pi_A^{el} - \pi_A^l$ is decreasing and $\pi_B^{el} + L^{el} - (\pi_B^l + L^l)$ is increasing in t. This is because when t is large, the marginal revenue in the foreign market is also large relative to the domestic market, and Firm A's incentive to supply goods to the domestic market becomes small. Further, a reduction of the license fee because of a decrease in Firm B's profit, including the license fee, owing to Firm A's entry into the domestic market becomes small. Their sum is positive when t is very small ($t < t^*$), and negative when t is not so small. Therefore, license with entry is the optimal strategy when t is very small and license without entry is the optimal strategy when t is not so small.

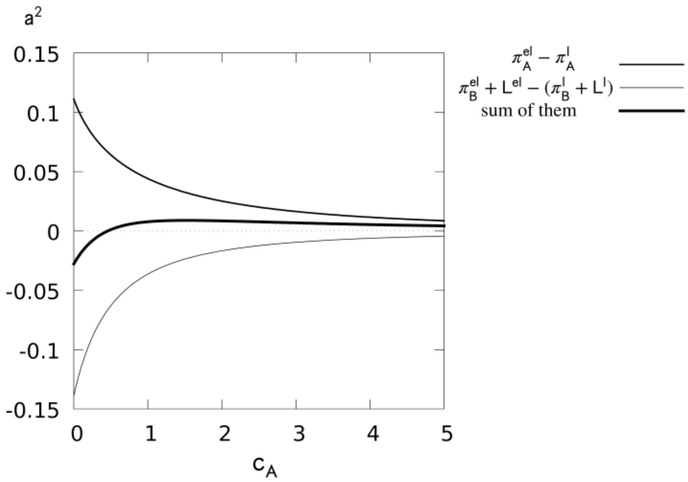

Figure 8. Relationship among $\pi_A^{el} - \pi_A^l$, $\pi_B^{el} + L^{el} - (\pi_B^l + L^l)$ and c_A

In Figure 8, we illustrate the relationship among $\pi_A^{el} - \pi_A^l$, $\pi_B^{el} + L^{el} - (\pi_B^l + L^l)$ and c_A, assuming $c_B = 5$ and $t = 0.3$. As we have pointed out, if t is larger than $2/3$, license without entry is the optimal strategy. Thus, in Figure 8, we consider a value of t such that both license with entry and license without entry strategy can be optimal strategies. It seems that $\pi_A^{el} - \pi_A^l$ is decreasing and $\pi_B^{el} + L^{el} - (\pi_B^l + L^l)$ is increasing in c_A. This is because when c_A is small, Firm A's profit from its own production when it enters the domestic market becomes large, and the reduction of the license fee due to the decrease in Firm B's profit, including the license fee owing to Firm A's entry into the domestic market, becomes large. The former effect is dominated by the latter effect when c_A is small. Thus, the optimal strategy is license without entry when c_A is small.

Summary of the Results

The results are summarized as follows.

1. When $t > (2c_B + 1)/c_A$ or $(2c_A + 1)/c_A < t \le (2c_B + 1)/c_A$, Firm A does not enter the domestic market, and its optimal strategy is license without entry.

2. When $t \le (2c_A + 1)/c_A$,

(1) Entry without license is never the optimal strategy for Firm A.

(2) If the ratio of the size of the foreign market to that of the domestic market is small, license with entry is the optimal strategy for Firm A.

(3) If the ratio of the size of the foreign market to that of the domestic market is not small, license without entry is the optimal strategy for Firm A.

(4) If c_A is small, ($c_A \le (\sqrt{2} - 1)/2$). That is, the cost advantage is significant and license without entry is always the optimal strategy.

6 Effects of a Capacity Limit and Transportation Cost

6.1 Capacity Limit

Suppose that Firm A encounters a strict capacity limit for its production. We assume that the output of Firm A when it does not enter the domestic market is not constrained by a capacity limit. A capacity limit may be effective when Firm A enters the domestic market with or without a license to Firm B. We consider the example of a linear demand function and a quadratic cost function from Section 5. We consider Case 3, in which Firm A enters the domestic market even when it sells a license, and so the relative size of the foreign market, t, is smaller than $(2c_A + 1)/c_A$. Let us denote the case of entry with license as Case EL, the case of license without entry as Case L, and the case of entry without license as Case E. Further, let us denote the total output of Firm A in each case as X_A^{el}, X_A^l, and X_A^e, respectively. Then, from the calculation results in Section 5 without capacity limit we have

$$X_A^{el} = x_A + y_A = \frac{a(4c_A t + 3t + 4c_A + 2)}{2(4c_A^2 t + 3c_A t + 4c_A^2 + 8c_A + 3)},$$

$$X_A^l = y_A = \frac{at}{2(c_A t + 1)},$$

$$X_A^e = x_A + y_A = \frac{a(4c_B t + 3t + 4c_B + 2)}{2(4c_A c_B t + 3c_A t + 4c_A c_B + 4c_B + 4c_A + 3)}.$$

These are increasing in t because

$$\frac{dX_A^l}{dt} = \frac{a}{2(c_A t+1)^2} > 0,$$

$$\frac{dX_A^{el}}{dt} = \frac{3a(2c_A + 1)(4c_A + 3)}{2(4c_A^2 t + 3c_A t + 4c_A^2 + 8c_A + 3)^2} > 0,$$

$$\frac{dX_A^e}{dt} = \frac{a(4c_B + 3)(4c_B + 2c_A + 3)}{2(4c_A c_B t + 3c_A t + 4c_A c_B + 4c_B + 4c_A + 3)^2} > 0.$$

Comparing these total outputs, we obtain

$$X_A^{el} - X_A^l = \frac{a(2c_A + 1 - c_A t)}{(c_A t + 1)(4c_A^2 t + 3c_A t + 4c_A^2 + 8c_A + 3)},$$

$$X_A^e - X_A^l = \frac{a(2c_B + 1 - c_A t)}{(c_A t + 1)(4c_A c_B t + 3c_A t + 4c_A c_B + 4c_B + 4c_A + 3)},$$

$$X_A^e - X_A^{el} = \frac{2a(c_B - c_A)(2c_A t + 2c_A + 1)}{(4c_A^2 t + 3c_A t + 4c_A^2 + 8c_A + 3)(4c_A c_B t + 3c_A t + 4c_A c_B + 4c_B + 4c_A + 3)}$$

They are positive when $t < (2c_A + 1)/c_A$, and $X_A^{el} - X_A^l = 0$ when $t = (2c_A + 1)/c_A$. We assume that the capacity of Firm A is equal to the output in Case L when $t = (2c_A + 1)/c_A$. It is denoted by $\bar{X}_A$. We have $\bar{X}_A = a(2c_A + 1)/4c_A(c_A + 1)$. Let

$$t = \frac{4c_A c_B + 4c_B + 4c_A^2 + 6c_A + 3}{c_A(4c_B + 3)}.$$

$X_A^e = a(2c_A + 1)/4c_A(c_A + 1)$ when $t = \tilde{t}$. We see $\tilde{t} - (2c_A + 1)/c_A = -4(c_B - c_A)/4c_B + 3 < 0$. The capacity limit in Case E is effective if $\tilde{t} < t < 2c_A + 1/c_A$.

Under the capacity limit, the problem of profit maximization for Firm A in Case E is as follows.

$$\max_{x_A, y_A}(a - x_A - x_B)x_A + (a - \frac{y_A}{t})y_A - c_A(x_A + y_A)^2$$
$$+\theta(x_A + y_A - \bar{X}_A).$$

θ is the Lagrange multiplier. The conditions for profit maximization are

$$a - 2x_A - x_B - 2c_A(x_A + y_A) = a - \frac{2}{t}y_A - 2c_A(x_A + y_A),$$
$$x_A + y_A = \bar{X}_A.$$

Then, we get

$$2x_A + x_B = \frac{2}{t}y_A.$$

With the profit maximization condition for Firm B, we obtain the equilibrium profit of Firm A in Case E under the capacity limit as follows.

$$\pi_A^e = \frac{a^2\lambda_J}{16ca^2(ca+1)^2(4cbt+3t+4cb+4)^2}. \tag{6}$$

The profit of Firm B in Case E under the capacity limit is different from that without the capacity limit. Then, the license fee in Case EL under the capacity limit is different from that without the capacity limit, though the capacity limit does not affect the equilibrium in Case EL. The profit of Firm B in Case E under the capacity limit is

$$\pi_B^e = \frac{a^2(c_B+1)(4c_A^2t+4c_At+4c_A^2+2c_A-1)^2}{4c_A^2(c_A+1)^2(4c_Bt+3t+4c_B+4)^2}$$

The license fee and the total profit of Firm A in Case EL under the capacity limit are

$$L^{el} = \frac{a^2\lambda_K}{4c_A^2(c_A+1)^2(4c_A^2t+3c_At+4c_A^2+8c_A+3)^2(4c_Bt+3t+4c_B+4)^2}.$$

$$\pi_A^{el}+L^{el}=\frac{a^2\lambda_L}{4c_A^2(c_A+1)^2(4c_A^2t+3c_At+4c_A^2+8c_A+3)^2(4c_Bt+3t+4c_B+4)^2}. \quad (7)$$

Comparing (6) and (7) yields

$$\pi_A^{el}+L^{el}-\pi_A^e$$
$$=\frac{a^2\lambda_M}{16c_A^2(c_A+1)^2(4c_A^2t+3c_At+4c_A^2+8c_A+3)^2(4c_Bt+3t+4c_B+4)^2}.$$

In Figure 9, we depict examples of $\pi_A^{el}+L^{el}-\pi_A^e$ assuming that $c_B=5$ for $c_A=1,2,$ and 4. In Figure 10, examples of $\pi_A^{el}+L^{el}-\pi_A^e$ assuming that $c_B=10$ for $c_A=1,4,$ and 8 are illustrated. When $c_A=1$, $(2c_A+1)/c_A=3$; when $c_A=2$, $(2c_A+1)/c_A=2.5$; when $c_A=4$, $(2c_A+1)/c_A=2.25$; and when $c_A=8$, $(2c_A+1)/c_A=2.125$. When $c_B=5$, $\tilde{t}\approx 2.3$ for $c_A=1$; $\tilde{t}\approx 1.98$ for $c_A=2$; and $\tilde{t}\approx 2.08$ for $c_A=4$. When $c_B=10$, $\tilde{t}\approx 2.17$ for $c_A=1$; $\tilde{t}=1.69$ for $c_A=4$; and $\tilde{t}\approx 1.94$ for $c_A=8$. These figures indicate that entry without license is not the optimal strategy.

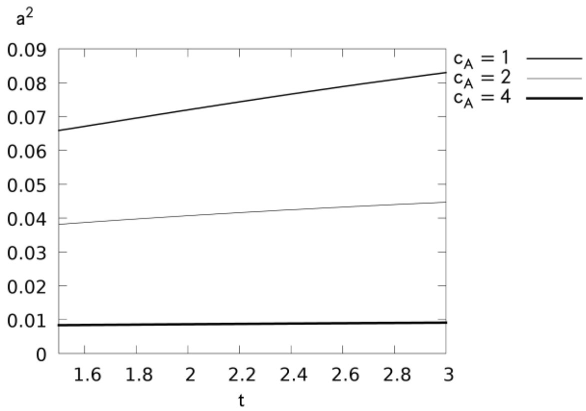

Figure 9. Illustration of $\pi_A^{el}+L^{el}-\pi_A^e$: $c_B=5$

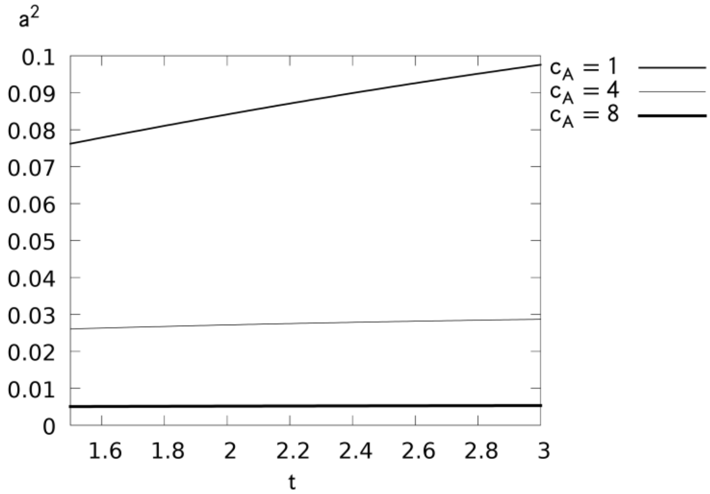

Figure 10. Illustration of $\pi_A^{el} + L^{el} - \pi_A^{e}$: $c_B = 10$

The capacity limit does not affect the equilibrium of Case EL and Case L. The effect of the capacity limit on Firm B's profit in Case E commonly affects the license fee in Case L and Case EL. Therefore, the presence of the capacity limit does not affect the optimal strategy of Firm A in this example. An analysis of a more general case with a capacity constraint is a theme for future research.

6.2 Transportation Cost

We sketch the effects of transportation cost on Firm A's strategy choice. Now, we assume that Firm A produces its good in the foreign country and as such, it must incur transportation costs to sell the good in the domestic market. Let *r* be Firm A's transportation cost per supply of the good exported to the domestic market. We use the example of the linear demand function and convex cost function from Section 5. Then, Firm A's equilibrium supply to the domestic market in the case of entry without license and that in the case of license with entry are as follows.

1. Entry without license

$$x_A = -\frac{a + 2ac_B - 2c_Ac_Brt - 2c_Art - ac_At - 2c_Br - 2r}{4c_Ac_Bt + 3c_At + 4c_Ac_B + 4c_B + 4c_A + 3}.$$

When $t > a(2c_B + 1) - 2(c_B + 1)r/2c_A(c_B + 1)r + ac_A,\ x_A = 0.$
As $a(2c_B + 1) - 2(c_B + 1)r/2c_A(c_B + 1)r + ac_A < (2c_B + 1)/c_A$, the larger the transportation cost, the more likely it is that Firm A will not enter the domestic market without license.

2. License with entry

$$x_A = \frac{a + 2ac_A - 2c_A^2 rt - 2c_A rt - ac_A t - 2c_A r - 2r}{4c_A^2 t + 3c_A t + 4c_A^2 + 8c_A + 3}.$$

When $t > a(2c_A + 1) - 2(c_A + 1)r/2c_A(c_A + 1)r + ac_A,\ x_A = 0.$
As $a(2c_A + 1) - 2(c_A + 1)r/2c_A(c_A + 1)r + ac_A < (2c_A + 1)/c_A$, the larger the transportation cost, the more likely it is that Firm A will not enter the domestic market when it sells a license.

We present the analyses of the optimal strategies for Firm A using a graphical representation because the quantitative results are very complicated. The details of the calculation are available upon request.

The difference between Firm A's total profit (the sum of sales profit and license fee) in the case of license without entry and its total profit in the case of license with entry is illustrated in Figure 11, for various values of t assuming $c_A = 2$, $c_B = 5$, and $a = 10$. The difference between Firm A's total profit in the case of license without entry and its total profit in the case of entry without license is illustrated in Figure 12, for various values of t assuming $c_A = 2$, $c_B = 5$, and $a = 10$. Figure 12 indicates that entry without license is never the optimal strategy. The transportation cost reduces the profit of both the entry without license and license without entry strategies. In case of the latter strategy, the transportation cost reduces the threat of entry by Firm A as well as the license fee. On the other hand, Figure 11 indicates that the relationship between the optimal strategy and the transportation cost is not straightforward. This is because Firm A's supply to the domestic market is decreasing and its supply to the foreign market is increasing with respect to the transportation cost. Figure 13 illustrates such a situation assuming $c_A = 2$ and $c_B = 5$, $a = 10$, and $t = 0.5$.

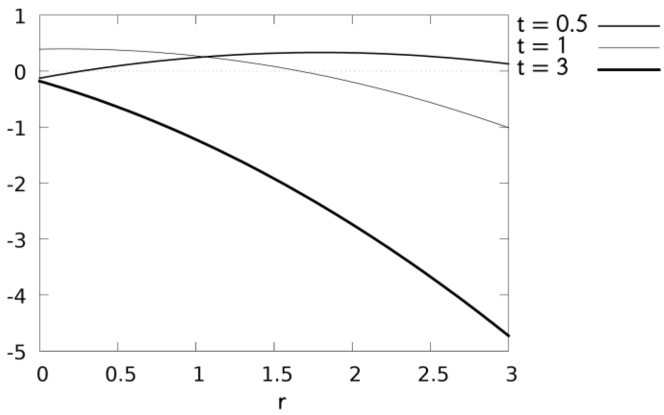

Figure 11. Illustration of $\boldsymbol{\pi_A^l + L^l - (\pi_A^{el} + L^{el})}$

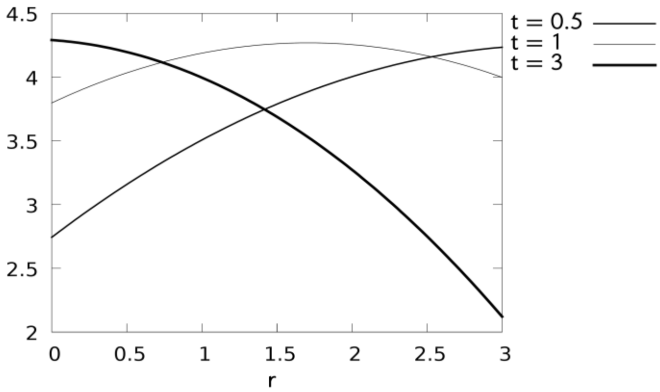

Figure 12. Illustration of $\boldsymbol{\pi_A^l + L^l - \pi_A^e}$

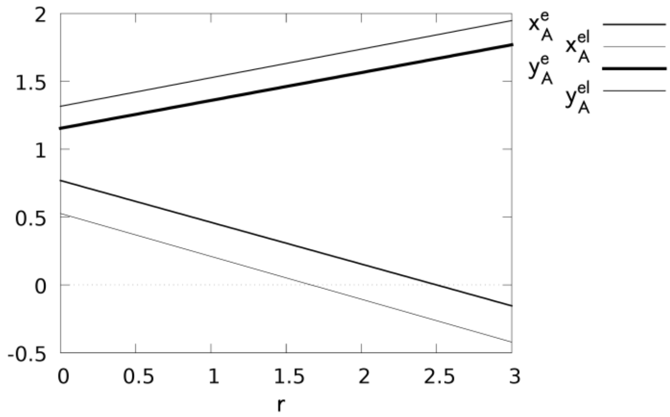

Figure 13. Illustration of x_A^e, x_A^{el}, y_A^e, **and** y_A^{el}

7 Concluding Remarks

We have examined the optimal strategies for a foreign firm with cost advantage in an international duopoly when it can enter the domestic market. Further, we have demonstrated that its optimal strategy depends on the relative size of the foreign and domestic markets. If the foreign market is large relative to the domestic market, license without entry is the optimal strategy for the foreign firm. On the other hand, if the foreign market is relatively small, license with entry may be the optimal strategy. In future research, we want to analyze the problem under general demand and cost functions, and an oligopolistic situation with more than two firms. In this paper, we only considered licensing with a fixed license fee. However, a licensor may sell a license based on royalty per output with a fixed license fee. Then, one of which may be negative. We plan to study a situation where a licensor in a foreign country sells a license of its cost reducing technology to a domestic firm for a royalty with a fixed license fee.

Acknowledgment

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Appendix: Details of Calculations

$$\lambda_A = 16c_A c_B^2 t^2 + 24c_A c_B t^2 + 4c_A^2 t^2 + 9c_A t^2 + 32c_A c_B^2 t + 16c_B^2 t + 40c_A c_B t + 24c_B + 4c_A^2 t + 16c_A t + 9t + 16c_A c_B^2 + 16c_B^2 + 16c_A c_B + 16c_B + 4c_A + 4.$$

$$\lambda_B = 16c_A^2 c_B^2 t^2 - 16c_A^3 c_B t^2 + 8c_A^2 c_B t^2 - 16c_A^3 t^2 - 7c_A^2 t^2 + 32c_A^2 c_B^2 t + 32c_A c_B^2 t - 32c_A^3 c_B t + 8c_A^2 c_B t + 32c_A c_B t - 32c_A^3 t - 24c_A^2 t + 2c_A t + 16c_A^2 c_B^2 + 32c_A c_B^2 + 16c_B^2 - 16c_A^3 c_B + 36c_A c_B + 20c_B - 16c_A^3 - 16c_A^2 + 4c_A + 5.$$

$$\lambda_C = 32c_A^3 c_B^2 t^3 + 16c_A^2 c_B^2 t^3 - 16c_A^4 c_B t^3 + 32c_A^3 c_B t^3 + 24c_A^2 c_B t^3 - 16c_A^4 t^3 + 2c_A^3 t^3 + 9c_A^2 t^3 + 64c_A^3 c_B^2 t^2 + 112c_A^2 c_B^2 t^2 + 32c_A c_B^2 t^2 - 32c_A^4 c_B t^2 + 48c_A^3 c_B t^2 + 144c_A^2 c_B t^2 + 48c_A c_B t^2 - 32c_A^4 t^2 - 16c_A^3 t^2 + 37c_A^2 t^2 + 18c_A t^2 + 32c_A^3 c_B^2 t + 112c_A^2 c_B^2 t + 96c_A c_B^2 t + 16c_B^2 t - 16c_A^4 c_B t + 132c_A^2 c_B t + 132c_A c_B t + 24c_B t - 16c_A^4 t - 32c_A^3 t + 20c_A^2 t + 40c_A t + 9t + 16c_A^2 c_B^2 + 32c_A c_B^2 + 16c_B^2 - 16c_{\mathrm{A}}^3 c_B + 36c_A c_B + 20c_B - 16c_A^3 - 16c_A^2 + 4c_A + 5.$$

$$\lambda_D = 16c_A^3 t^2 + 28c_A^2 t^2 + 9c_A t^2 + 32c_A^3 t + 60c_A^2 t + 40c_A t + 9t + 16c_A^3 + 32c_A^2 + 20c_A + 4$$

$$\lambda_E = 16c_A^3 c_B t^2 + 16c_A^2 c_B t^2 + 16c_A^3 t^2 + 15c_A^2 t^2 + 32c_A^3 c_B t + 64c_A^2 c_B t + 32c_A c_B t + 32c_A^3 t + 64c_A^2 t + 30c_A t + 16c_A^3 c_B + 48c_A^2 c_B + 48c_A c_B + 16c_B + 16c_A^3 + 48c_A^2 + 48c_A + 15.$$

$$\lambda_F = 512c_A^5c_B^2t^4 + 704c_A^4c_B^2t^4 + 144c_A^3c_B^2t^4 - 256c_A^6c_Bt^4 + 384c_A^5c_Bt^4 + 912c_A^4c_Bt^4 + 216c_A^3c_Bt^4 - 256c_A^6t^4 - 96c_A^5t^4 + 252c_A^4t^4 + 81c_A^3t^4 + 2048c_A^5c_B^2t^3 + 4160c_A^4c_B^2t^3 + 2592c_A^3c_B^2t^3 + 432c_A^2c_B^2t^3 - 1024c_A^6c_Bt^3 + 896c_A^5c_Bt^3 + 4768c_A^4c_Bt^3 + 3528c_A^3c_Bt^3 + 648c_A^2c_Bt^3 - 1024c_A^6t^3 - 1088c_A^5t^3 + 780c_A^4t^3 + 1080c_A^3t^3 + 243c_A^2t^3 + 3072c_A^5c_B^2t^2 + 8256c_A^4c_B^2t^2 + 7952c_A^3c_B^2t^2 + 3200c_A^2c_B^2t^2 + 432c_Ac_B^2t^2 - 1536c_A^6c_Bt^2 + 384c_A^5c_Bt^2 + 8144c_A^4c_Bt^2 + 10064c_A^3c_Bt^2 + 4476c_A^2c_Bt^2 + 648c_Ac_Bt^2 - 1536c_A^6t^2 - 2656c_A^5t^2 + 80c_A^4t^2 + 2400c_A^3t^2 + 1440c_A^2t^2 + 243c_At^2 + 2048c_A^5c_B^2t + 6848c_A^4c_B^2t + 8704c_A^3c_B^2t + 5200c_A^2c_B^2t + 1440c_Ac_B^2t + 144c_B^2t - 1024c_A^6c_Bt - 384c_A^5c_Bt + 5632c_A^4c_Bt + 9952c_A^3c_Bt + 6752c_A^2c_Bt + 2016c_Ac_Bt + 216c_Bt - 1024c_A^6t - 2432c_A^5t - 1152c_A^4t + 1440c_A^3t + 1740c_A^2t + 648c_At + 81t + 512c_A^5c_B^2 + 2048c_A^4c_B^2 + 3200c_A^3c_B^2 + 2432c_A^2c_B^2 + 896c_Ac_B^2 + 128c_B^2 - 256c_A^6c_B - 256c_A^5c_B + 1344c_A^4c_B + 3200c_A^3c_B + 2768c_A^2c_B + 1072c_Ac_B + 156c_B - 256c_A^6 - 768c_A^5 - 704c_A^4 + 32c_A^3 + 400c_A^2 + 216c_A + 36.$$

$$\lambda_G = 16c_A^3c_B^2t^3 - 16c_A^4c_Bt^3 + 8c_A^3c_Bt^3 - 20c_A^4t^3 - 11c_A^3t^3 + 32c_A^3c_B^2t^2 + 48c_A^2c_B^2t^2 - 32c_A^4c_Bt^2 + 8c_A^3c_Bt^2 + 56c_A^2c_Bt^2 - 36c_A^4t^2 - 40c_A^3t^2 - c_A^2t^2 + 16c_A^3c_B^2t + 48c_A^2c_B^2t + 32c_Ac_B^2t - 16c_A^4c_Bt - 16c_A^3c_Bt + 60c_A^2c_Bt + 52c_Ac_Bt - 16c_A^4\mathrm{t} - 40c_A^3t - 8c_A^2t + 11c_At - 16c_A^3c_B - 16c_A^2c_B + 4c_Ac_B + 4c_B - 16c_A^3 - 20c_A^2 - 4c_A + 1.$$

$$\lambda_H = 32c_A^4c_Bt^3 + 40c_A^3c_Bt^3 + 40c_A^4t^3 + 42c_A^3t^3 + 96c_A^4c_Bt^2 + 160c_A^3c_Bt^2 + 76c_A^2c_Bt^2 + 112c_A^4t^2 + 198c_A^3t^2 + 87c_A^2t^2 + 96c_A^4c_Bt + 200\mathrm{c}_A^3c_Bt + 132c_A^2c_Bt + 32c_Ac_Bt + 104c_A^4t + 244c_A^3t + 188c_A^2t + 48c_At + 32c_A^4c_B + 80c_A^3c_B + 56c_A^2c_B + 4c_Ac_B - 4c_B + 32c_A^4 + 88c_A^3 + 84c_A^2 + 30c_A + 3.$$

$$\lambda_I = 12c_A^3t^2 + 11c_A^2t^2 + 4c_A^3t + 18c_A^2t + 12c_At - 8c_A^3 - 12c_A^2 - 2c_A + 1.$$

$$\lambda_J = 64c_A^3c_B^2t^2 + 128c_A^2c_B^2t^2 + 48c_Ac_B^2t^2 + 96c_A^3c_Bt^2 + 192c_A^2c_Bt^2 + 72c_Ac_Bt^2 + 16c_A^4t^2 + 68c_A^3t^2 + 88c_A^2t^2 + 27c_At^2 + 128c_A^3c_B^2t + 192c_A^2c_B^2t + 32c_Ac_B^2t - 16c_B^2t + 160c_A^3c_Bt + 256c_A^2c_Bt + 40c_Ac_Bt - 24c_Bt + 16c_A^4t + 64c_A^3t + 76c_A^2t + 4c_At - 9t + 64c_A^3c_B^2 + 64c_A^2c_B^2 - 16c_Ac_B^2 - 16c_B^2 + 64c_A^3c_B + 64c_A^2c_B - 32c_Ac_B - 24c_B - 16c_A - 8.$$

$$\lambda_K = 256c_A^7c_B^2t^4 + 768c_A^6c_B^2t^4 + 768c_A^5c_B^2t^4 + 256c_A^4c_B^2t^4 - 256c_A^8c_Bt^4 - 512c_A^7c_Bt^4 - 16c_A^6c_Bt^4 + 480c_A^5c_Bt^4 + 240c_A^4c_Bt^4 - 256c_A^8t^4 - 752c_A^7t^4 - 736c_A^6t^4 - 240c_A^5t^4 + 1024c_A^7c_B^2t^3 + 3328c_A^6c_B^2t^3 + 3840c_A^5c_B^2t^3 + 1792c_A^4c_B^2t^3 + 256c_A^3c_B^2t^3 - 1024c_A^8c_Bt^3 - 2304c_A^7c_Bt^3 - 672c_A^6c_Bt^3 + 1648c_A^5c_Bt^3 + 1208c_A^4c_Bt^3 + 168c_A^3c_Bt^3 - 1024c_A^8t^3 - 3296c_A^7t^3 - 3888c_A^6t^3 - 2048c_A^5t^3 - 504c_A^4t^3 - 72c_A^3t^3 + 1536c_A^7c_B^2t^2 + 5376c_A^6c_B^2t^2 + 6976c_A^5c_B^2t^2 + 4032c_A^4c_B^2t^2 + 960c_A^3c_B^2t^2 + 64c_A^2c_B^2t^2 - 1536c_A^8c_Bt^2 - 3840c_A^7c_Bt^2 - 1616c_A^6c_Bt^2 + 2928c_A^5c_Bt^2 + 3108c_A^4c_Bt^2 + 956c_A^3c_Bt^2 + 87c_A^2c_Bt^2 - 1536c_A^8t^2 - 5360c_A^7t^2 - 6928c_A^6t^2 - 3948c_A^5t^2 - 848c_A^4t^2 + 24c_A^3t^2 + 27c_A^2t^2 + 1024c_A^7c_B^2t + 3840c_A^6c_B^2t + 5504c_A^5c_B^2t + 3712c_A^4c_B^2t + 1152c_A^3c_B^2t + 128c_A^2c_B^2t - 1024c_A^8c_Bt - 2816c_A^7c_Bt - 1280c_A^6c_Bt + 3104c_A^5c_Bt + 4352c_A^4c_Bt + 2240c_A^3c_Bt + 536c_A^2c_Bt + 54c_Ac_Bt - 1024c_A^8t - 3840c_A^7t - 5120c_A^6t - 2400c_A^5t + 640c_A^4t + 1088c_A^3t + 408c_A^2t + 54c_At + 256c_A^7c_B^2 + 1024c_A^6c_B^2 + 1600c_A^5c_B^2 + 1216c_A^4c_B^2 + 448c_A^3c_B^2 + 64c_A^2c_B^2 - 256c_A^8c_B - 768c_A^7c_B - 320c_A^6c_B + 1344c_A^5c_B + 2112c_A^4c_B + 1232c_A^3c_B + 268c_A^2c_B - 12c_Ac_B - 9c_B - 256c_A^8 - 1024c_A^7 - 1344c_A^6 - 256c_A^5 + 896c_A^4 + 784c_A^3 + 204c_A^2 - 12c_A - 9.$$

$$\begin{aligned}\lambda_L = {} & 512c_A^7c_B^2t^4 + 1728c_A^6c_B^2t^4 + 2064c_A^5c_B^2t^4 + 992c_A^4c_B^2t^4 + 144c_A^3c_B^2t^4 \\ & - 256c_A^8c_Bt^4 - 128c_A^7c_Bt^4 + 1424c_A^6c_Bt^4 + 2424c_A^5c_Bt^4 \\ & + 1344c_A^4c_Bt^4 + 216c_A^3c_Bt^4 - 256c_A^8t^4 - 608c_A^7t^4 - 196c_A^6t^4 \\ & + 489c_A^5t^4 + 414c_A^4t^4 + 81c_A^3t^4 + 2048c_A^7c_B^2t^3 + 7232c_A^6c_B^2t^3 \\ & + 9504c_A^5c_B^2t^3 + 5648c_A^4c_B^2t^3 + 1472c_A^3c_B^2t^3 + 144c_A^2c_B^2t^3 \\ & - 1024c_A^8c_Bt^3 - 640c_A^7c_Bt^3 + 5664c_A^6c_Bt^3 + 10792c_A^5c_Bt^3 \\ & + 7360c_A^4c_Bt^3 + 2064c_A^3c_Bt^3 + 216c_A^2c_Bt^3 - 1024c_A^8t^3 \\ & - 2624c_A^7t^3 - 1332c_A^6t^3 + 1624c_A^5t^3 + 1941c_A^4t^3 + 666c_A^3t^3 \\ & + 81c_A^2t^3 + 3072c_A^7c_B^2t^2 + 11328c_A^6c_B^2t^2 + 16016c_A^5c_B^2t^2 \\ & + 10752c_A^4c_B^2t^2 + 3408c_A^3c_B^2t^2 + 416c_A^2c_B^2t^2 - 1536c_A^8c_Bt^2 \\ & - 1152c_A^7c_Bt^2 + 8784c_A^6c_Bt^2 + 18672c_A^5c_Bt^2 + 14748c_A^4c_Bt^2 \\ & + 5164c_A^3c_Bt^2 + 687c_A^2c_Bt^2 - 1536c_A^8t^2 - 4192c_A^7t^2 \\ & - 2416c_A^6t^2 + 2856c_A^5t^2 + 4148c_A^4t^2 + 1812c_A^3t^2 + 279c_A^2t^2 \\ & + 2048c_A^7c_B^2t + 7872c_A^6c_B^2t + 11776c_A^5c_B^2t + 8528c_A^4c_B^2t \\ & + 2976c_A^3c_B^2t + 400c_A^2c_B^2t - 1024c_A^8c_Bt - 896c_A^7c_Bt \\ & + 6272c_A^6c_Bt + 14848c_A^5c_Bt + 13376c_A^4c_Bt + 5664c_A^3c_Bt \\ & + 1048c_A^2c_Bt + 54c_Ac_Bt - 1024c_A^8t - 2944c_A^7t - 1600c_A^6t \\ & + 3072c_A^5t + 4848c_A^4t + 2688c_A^3t + 648c_A^2t + 54c_At + 512c_A^7c_B^2 \\ & + 2048c_A^6c_B^2 + 3200c_A^5c_B^2 + 2432c_A^4c_B^2 + 896c_A^3c_B^2 + 128c_A^2c_B^2 \\ & - 256c_A^8c_B - 256c_A^7c_B + 1728c_A^6c_B + 4544c_A^5c_B + 4544c_A^4c_B \\ & + 2128c_A^3c_B + 396c_A^2c_B - 12c_Ac_B - 9c_B - 256c_A^8 - 768c_A^7 \\ & - 320c_A^6 + 1344c_A^5 + 2112c_A^4 + 1232c_A^3 + 268c_A^2 - 12c_A - 9.\end{aligned}$$

$$\begin{aligned}\lambda_M = {} & 1024c_A^7c_B^2t^4 + 3328c_A^6c_B^2t^4 + 3840c_A^5c_B^2t^4 + 1664c_A^4c_B^2t^4 + 144c_A^3c_B^2t^4 \\ & - 1024c_A^8c_Bt^4 - 2048c_A^7c_Bt^4 + 320c_A^6c_Bt^4 + 3072c_A^5c_Bt^4 \\ & + 1920c_A^4c_Bt^4 + 216c_A^3c_Bt^4 - 1280c_A^8t^4 - 3904c_A^7t^4 \\ & - 3968c_A^6t^4 - 1200c_A^5t^4 + 216c_A^4t^4 + 81c_A^3t^4 + 4096c_A^7c_B^2t^3 \\ & + 13056c_A^6c_B^2t^3 + 14336c_A^5c_B^2t^3 + 5760c_A^4c_B^2t^3 + 224c_A^3c_B^2t^3 \\ & - 144c_A^2c_B^2t^3 - 4096c_A^8c_Bt^3 - 8192c_A^7c_Bt^3 + 128c_A^6c_Bt^3 \\ & + 8832c_A^5c_Bt^3 + 4672c_A^4c_Bt^3 - 168c_A^3c_Bt^3 - 216c_A^2c_Bt^3 \\ & - 4864c_A^8t^3 - 15488c_A^7t^3 - 18176c_A^6t^3 - 9760c_A^5t^3 \\ & - 2808c_A^4t^3 - 684c_A^3t^3 - 81c_A^2t^3 + 6144c_A^7c_B^2t^2 \\ & + 19200c_A^6c_B^2t^2 + 20480c_A^5c_B^2t^2 + 7232c_A^4c_B^2t^2 - 1136c_A^3c_B^2t^2 \\ & - 1072c_A^2c_B^2t^2 - 144c_Ac_B^2t^2 - 6144c_A^8c_Bt^2 - 12288c_A^7c_Bt^2 \\ & + 1088c_A^6c_Bt^2 + 15872c_A^5c_Bt^2 + 9392c_A^4c_Bt^2 - 272c_A^3c_Bt^2 \\ & - 1212c_A^2c_Bt^2 - 216c_Ac_Bt^2 - 6912c_A^8t^2 - 22336c_A^7t^2 \\ & - 26048c_A^6t^2 - 12848c_A^5t^2 - 2464c_A^4t^2 - 492c_A^3t^2 - 324c_A^2t^2 \\ & - 81c_At^2 + 4096c_A^7c_B^2t + 12544c_A^6c_B^2t + 13312c_A^5c_B^2t \\ & + 5440c_A^4c_B^2t + 1152c_A^3c_B^2t + 1184c_A^2c_B^2t + 768c_Ac_B^2t + 144c_B^2t \\ & - 4096c_A^8c_Bt - 8192c_A^7c_Bt + 3072c_A^6c_Bt + 19072c_A^5c_Bt \\ & + 18944c_A^4c_Bt + 10208c_A^3c_Bt + 4384c_A^2c_Bt + 1440c_Ac_Bt \\ & + 216c_Bt - 4352c_A^8t - 13824c_A^7t - 13120c_A^6t + 1472c_A^5t \\ & + 11040c_A^4t + 8608c_A^3t + 3372c_A^2t + 756c_At + 81t \\ & + 1024c_A^7c_B^2 + 3072c_A^6c_B^2 + 3328c_A^5c_B^2 + 2304c_A^4c_B^2 + 2368c_A^3c_B^2 \\ & + 2112c_A^2c_B^2 + 912c_Ac_B^2 + 144c_B^2 - 1024c_A^8c_B - 2048c_A^7c_B \\ & + 1792c_A^6c_B + 8960c_A^5c_B + 11904c_A^4c_B + 9216c_A^3c_B \\ & + 4656c_A^2c_B + 1392c_Ac_B + 180c_B - 1024c_A^8 - 3072c_A^7 \\ & - 1280c_A^6 + 5632c_A^5 + 9600c_A^4 + 6848c_A^3 + 2544c_A^2 + 480c_A \\ & + 36.\end{aligned}$$

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