The Effects of Unionization in an R&D Growth Model with (In)determinate Equilibrium

Chung-Hui Lai[*]

Department of Accounting, Feng Chia University, Taiwan

Chen-Sheng Yang

Department of International Business, Central Taiwan University of Science and Technology, Taiwan

This paper extends an R&D-based growth model of the Rivera-Batiz and Romer-type [Quarterly Journal of Economics 106 (1991) 531] endogenous growth model by embodying a union with elastic labor to investigate the effects of unionization on employment and growth by highlighting the essence of internal conflict within the union. It is shown that an increase in the union's bargaining power or a union which is more employment-oriented boosts employment and economic growth when the balanced growth equilibrium is determinate. On the other hand, if the union is more wage-oriented, employment and economic growth are enhanced when the balanced growth equilibrium is indeterminate.

Keywords: union, collective bargaining, R&D, indeterminacy, endogenous growth
JEL classification: J50, O30, O40

1 Introduction

In continental Europe, European countries are characterized by relatively strong labor unions, especially in Belgium, Austria and the Scandinavian countries, in which the labor union participation rate is in the range of 70-90%. (Layard *et al*., 1991; Booth, 1995). By a union means that the wage is determined by bargaining

---

[*]Correspondence to: Email: p9217770@gmail.com. Address: Department of Accounting, Feng Chia University, No. 100 Wenhwa Rd., Seatwen, Taichung 40724, Taiwan.

and approximately 90% of wage contracts are determined through union bargaining in continental Europe. Unions play an important role in many high growth countries and therefore it is instructive to examine the relationship between unionization and the economic growth rate.

Unionization will increase the wage above the non-unionized level. High wages increase the revenue from introducing a labor-saving technology and thus spur R&D. On the contrary, firms argue that the wage increase decreases the profit from an innovation. Hence, there is an ambiguous effect in terms of the impact of union wage bargaining on the firm's incentive to invest in research. For example, Menezes-Fihlo and Van Reenen (2003) provide a broad survey of the empirical literature on the effect of unions and innovation and conclude that there is no consensus among these studies as to the way in which unions affect innovation. Lingens (2006) plots aggregate R&D expenditures and union wage coverage for a set of EU15 countries and the US and Japan that yields evidence of a negative relationship between investment in research and union wage bargaining. Ulph and Ulph (1994) show that the hold-up effect dominates in a right-to-manage setting. Haucap and Wey (2004) argue that unionization will foster the incentive to invest into research in the situation with centralized wage bargaining in which one union through bargaining obtains a uniform wage for the two duopolists. However, due to the focus on innovation, R&D has an important part to play in the endogenous growth model (Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992). The firms engage in R&D activities since they obtain the patent in order to acquire profits, and therefore promote economic growth. This paper introduces union-firm bargaining into an R&D-based endogenous growth model where firms can improve their productivity by means of R&D in order to investigate the relationship between unionization and innovation.

In recent years, economists have paid increasing attention to analyzing the effects of unionization on long-run economic growth. Palokangas (1996) and Lingens (2003) shed light on the role of intersectoral interaction and labor mobility in influencing the growth effect of unionization. Irmen and Wigger (2000) develop an OLG model that focuses on the role of intergenerational resource allocation and altruism in relation to that same effect. Boone (2000) demonstrates that the existence of a union dampens the growth rate of the economy. Chang *et al.* (2007) explore the

effects of unionization on unemployment, growth and welfare. Other papers analyzing the potential effects of unions on economic growth are Faini (1999), De Groot (2001), Bräuninger (2000), Palokangas (2004), and Lingens (2007).

Some studies which analyze the bargaining that takes place between unions and firms place the union in the final goods sector (Irmen and Wigger, 2000; Chang *et al.*, 2007), and the intermediate case of sectoral bargaining is not considered. Furthermore, some studies set it in the intermediate goods sector (Lingens, 2007; Palokangas, 1996). However, although they discuss the unionization effect in relation to the long-run growth effect, they do not investigate the indeterminacy of the equilibria, and that the endogeneity of labor supply can bring about fundamental changes in the stability of long-run equilibria (see Benhabib and Farmer, 1994) in the economic growth. More specifically, introducing an endogenous labor supply can change the dynamical system evaluated at the steady state.

Chen *et al.* (2017) mention that technological progress driven by innovation and R&D also acts as an important driver for growth, while capital accumulation is an important engine of economic growth.[1] Thus, in departing from their analysis, this paper sets up the Rivera-Batiz and Romer's type (1991) R&D-based endogenous growth model which takes heterogeneity into account with a union and endogenous labor supply, and uses it to explore the effects of unionization on economic growth by highlighting the essence of internal conflict within a union.

By setting up a Rivera-Batiz and Romer-type (1991) R&D-based endogenous growth model with elastic labor embodying a union, this paper explores the effects of unionization on economic growth and the stability of the long-run equilibrium. As for the bargaining framework, we follow Clark (1990) whereby both the union and the employer's federation bargain over the wage and employment through the generalized Nash bargaining solution. Given such a model, we show that there exist non-conflicting effects between the extent of the union being employment-oriented and being wage-oriented on the equilibrium level of employment and economic growth according to the stability properties of the economy. If the economy is indeterminate, a union that is increasingly wage-oriented raises the equilibrium level of employment and economic growth. If the economy is determinate, an increase in the union's bargaining power (or where the union becomes more

---

[1]See Aghion and Howitt (2009) for discussion on data from OECD countries.

employment-oriented) improves the equilibrium level of employment as well as the economic growth rate.

The remainder of this paper is organized as follows. Section 2 presents an R&D endogenous growth model with a union labor market. Section 3 analyzes the dynamics of the model and derives the conditions under which a balanced growth path is locally indeterminate. Section 4 analyzes the long-run effects of the unionization. Section 5 concludes the paper.

2   The Model

Consider a unionized economy that grows endogenously owing to its being driven by R&D with an endogenous labor supply. There are four types of agents in this economy: the final goods firms, the intermediate goods firms, the R&D firms and a labor union. The final goods firms produce the consumption goods using intermediate goods and labor. Each firm in the monopolistically competitive intermediate goods sector holds a blueprint, and uses this blueprint to produce one kind of product. In addition, we assume that any intermediate goods firm can meet the R&D cost needed to secure the net present value of profit associated with the new product developed. The R&D activity is assumed to involve free entry, and blueprints can be created only through the final goods used in research. The labor union exists in the final goods sector and its behavior reflects the internal conflict between its being employment-oriented and being wage-oriented rather than simply involving a rational maximization of choice. Households derive utility from consumption and leisure, and provide their labor elastically to firms.

2.1   Final Goods Firms, the Union, and Collective Bargaining

2.1.1   Final Goods Firms

Firms hire labor l and a continuum of intermediate capital goods x_i to produce the final output Y which can be consumed or invested. The production function of the final goods sector takes the following form:

$$Y_t = l_t^\beta \int_0^{n_t} x_{it}^\alpha di, \ \ 0 < \alpha < 1, \ \ 0 < \beta < 1, \tag{1}$$

where n_t represents the number of varieties of differentiated intermediate goods that expands over time due to technological progress. $i \in [0, n_t]$ is the range of intermediate goods existing at time t. In Eq. (1) we assume that the individual firm's production technology exhibits decreasing returns to scale in its internal intermediate goods x_{it} and labor l_t factors, i.e., $0 < \alpha + \beta < 1$. This implies that firms have a positive profit when the employer's federation has bargaining power. The total stock of producer durables is related to the aggregate capital stock $K_t = \int_0^{n_t} x_{it} di$.

Given the production function (1), the representative firm attempts to maximize its profit Π_t as follows:

$$\Pi = Y - wl - \int_0^n p_i x_i di, \tag{2}$$

where w and p_i are the wage rate and the price of intermediate goods in terms of the final goods, respectively.[2]

2.1.2 Union

Given the assumption of a closed shop union (Chang *et al.* (2007)), the union preferences are represented by the following modified Stone-Geary utility function:

$$U = (w - \overline{w})^{1-\upsilon} l^\upsilon, \ \ 0 < \upsilon < 1, \tag{3}$$

where U denotes the utility of the union, $\overline{w}$ is the competitive wage, and υ and $1 - \upsilon$ are the extent to which the union is employment-oriented and wage-oriented, respectively. The union is wage-oriented if υ is less than half and is employment-oriented if υ is greater than half.

---

[2]To simplify the notation, the time arguments will all be dropped.

2.1.3   Collective Bargaining

McDonald and Solow (1981) propose an important framework, the efficient bargaining model, in the trade union literature. The model's central feature is that the wage-employment contract negotiated by the employer and the union should be efficient. With this feature, we assume that both the union and the employers' federation bargain over wages and employment through a generalized Nash bargaining solution, subject to the final goods firms' demand for intermediate goods. This optimization problem can be expressed as:

$$\max_{w,l} \Omega = (U - \overline{U})^{\theta} \cdot (\Pi - \overline{\Pi})^{1-\theta} = [(w - \overline{w})^{1-\upsilon} l^{\upsilon}]^{\theta} \cdot [l^{\beta} \int_0^n x_i^{\alpha} di - wl - \int_0^n p_i x_i di]^{1-\theta}$$

$$\text{s.t.} \quad x_i = \arg\max_{x_i} \Pi,$$

where $\theta \in (0, 1)$ is the bargaining power of the union, $\overline{U}$ is the disagreement point of the union, and $\overline{\Pi}$ is the disagreement point of the final goods firm. We assume that the bargaining disagreement point results in a zero employment level. The impasse utility of both parties is zero, $\overline{U} = \overline{\Pi} = 0$.

By some simple manipulations, the optimal conditions for the wage and employment are given by:

$$w - \overline{w} = \frac{1-\upsilon}{\upsilon}\left[w - \beta l^{\beta-1} \int_0^n x_i^{\alpha} di \right], \tag{4}$$

$$w = \left[\beta + \frac{\theta \upsilon (1 - \alpha - \beta)}{1 - \theta(1 - \upsilon)} \right] l^{\beta-1} \int_0^n x_i^{\alpha} di, \tag{5}$$

and the intermediate goods' inverse demand function is:

$$p_i = \alpha l^{\beta} x_i^{\alpha-1}. \tag{6}$$

Eq. (5) describes the labor demand function. Given a particular level of employment, as the union's bargaining power θ increases, the negotiated wage rate will rise. According to Eq. (6), the rate of return on intermediate goods should be equal to the private marginal product of the intermediate goods.

Since the labor market is imperfect and characterized by unionization, this

implies that there is a positive profit for final goods firms. By substituting Eqs. (5)-(6) into Eq. (2), the representative final goods firm's profit function is given by:

$$\Pi = \frac{(1-\theta)(1-\alpha-\beta)}{1-\theta(1-\upsilon)} l^{\beta} \int_0^n x_i^{\alpha} di.$$

(7)

2.2 Intermediate Goods

The typical intermediate goods firm produces differentiated goods with a technology that requires one unit of capital per unit of intermediate goods ($x_i = k_i$). Given the prevailing rental rate r, each intermediate goods firm produces and sells a slightly unique variety of goods x_i to each final goods firm to maximize its profit:

$$\pi_i = (p_i - r)x_i$$

(8)

subject to the demand function (6). Profit maximization yields the following monopoly price:

$$p_i = p = \frac{r}{\alpha},$$

(9)

where the parameter $1/\alpha$ represents the intermediate goods firm's market power. Substituting Eq. (9) into Eq. (8), we obtain:

$$\pi_i = \frac{1-\alpha}{\alpha} r x_i.$$

(10)

2.3 R&D

R&D technology is such that, to develop a new idea, a researcher needs μ units of final goods to develop ideas.[3] The production function in the R&D sector is given by:

---

[3]Rivera-Batiz and Romer (1991) use this specification in the framework that they describe as the lab-equipment model of R&D.

$$\dot{n} = \frac{R}{\mu}, \tag{11}$$

where R is the amount of final goods devoted to R&D activities, $\dot{n}$ is the number of new blueprints created for a given period of time.

The research sector's profit flow is given by:

$$\pi_n = p_n \dot{n} - R, \tag{12}$$

where p_n represents the price of a new blueprint.

By substituting the production function, Eq. (11), into Eq. (12) and due to the property of perfect competition in the R&D sector ($\pi_n = 0$), the blueprint cost or value is as follows:

$$p_n = \mu. \tag{13}$$

Eq. (13) indicates that the value of the blueprint is equal to its cost.

Anyone can have free entry into the business of being an inventor as long as the R&D cost secures the net present value of the profit in intermediate goods, that is:

$$p_n = \int_t^\infty \pi(\omega) e^{-[1/(\omega-t)] \cdot \int_t^\omega r(\varpi)d\varpi} d\omega. \tag{14}$$

Differentiating the free entry condition in Eq. (14) with respect to time,[4] we obtain:

$$r = \frac{\pi}{p_n} + \frac{\dot{p}_n}{p_n}, \tag{15}$$

where π is the profit flow given by Eq. (10). Eq. (15) is a non-arbitrage condition which states that the rate of return on bonds, r, equals the rate of return on investing in R&D. The R&D rate of return equals the profit rate, π/p_n, plus the rate of capital gain or loss, $\dot{p}_n/p_n$.

Substituting Eqs. (10) and (13) into (15) and solving for x_i yields:

$$x_i = \bar{x} = \frac{\alpha\mu}{1-\alpha}. \tag{16}$$

Eq. (16) implies that the quantity of all intermediate goods is the same among these

---

[4]We use Leibniz's rule for the differentiation of a definite integral.

firms and is fixed through time.

2.4 Households

There is a continuum of identical infinitely lived households with one unit of time, each of which maximizes its lifetime utility:

$$\int_0^\infty V(c,l)e^{-\rho t}dt \equiv \int_0^\infty \frac{(c(1-l)^\eta)^{1-\sigma}}{1-\sigma}e^{-\rho t}dt, \tag{17}$$

where c is the individual's consumption, $0 \le l \le 1$ is the individual's labor supply, the positive parameter η denotes the weight on utility toward leisure, the positive parameter σ denotes the inverse of the intertemporal elasticity of substitution in consumption, and ρ denotes the subjective rate of time preference. The instantaneous utility function $V(c,l)$ is seen to be increasing in consumption and decreasing in labor supply at a decreasing rate, $V_c > 0$, $V_l < 0$, $V_{cc} < 0$ and $V_{ll} > 0$. Moreover, we assume that the utility function is concave in c and $1-l$, and this implies that $\sigma > \eta/(1+\eta)$.

The budget constraint faced by the representative household is given by:

$$\dot{k} + \dot{a} = ra + wl + \Pi + rk - c. \tag{18}$$

The value $a (= p_n n)$ denotes the value of equity shares of monopolistic firms.

The first-order conditions for this problem are given by

$$c^{-\sigma}(1-l)^{\eta(1-\sigma)} = \lambda, \tag{19}$$

$$c^{1-\sigma}\eta(1-l)^{\eta(1-\sigma)-1} = \lambda w, \tag{20}$$

$$\lambda r = -\dot{\lambda} + \rho\lambda, \tag{21}$$

given initial level of equity holdings, and the transversality condition $\lim_{t \leftarrow \infty} e^{-\rho t}\lambda k = 0$. λ stands for the shadow price of capital holdings. Dividing Eq. (20) by Eq. (19) results in:

$$\eta\frac{c}{1-l} = w. \tag{22}$$

Eq. (22) implies that the marginal rate of substitution between consumption and leisure is equal to the real wage rate at each point in time. In addition, it represents the labor supply function. From Eqs. (19) and (21), a simple manipulation with $\dot{l} = 0$ yields the standard Keynes-Ramsey rule:

$$\frac{\dot{c}}{c} = \frac{1}{\sigma}(r - \rho). \tag{23}$$

Since Eq. (23) is derived from the household's preference, we refer to it is the preference curve, or the consumption growth rate (Rivera-Batiz and Romer, 1991).

2.5 Equilibrium

In a symmetric equilibrium, all firms make the same choices, so that $x_i = \bar{x}$, $p_i = p$, $\pi_i = \pi$, and $K = n\bar{x}$. Equipped with this knowledge, we summarize the equilibrium condition of the economy as follows:

$$w = \left[\beta + \frac{\theta\upsilon(1-\alpha-\beta)}{1-\theta(1-\upsilon)} \right] l^{\beta-1} n\bar{x}^{\alpha}, \tag{5a}$$

$$p = \alpha l^{\beta}\bar{x}^{\alpha-1}, \tag{6a}$$

$$\Pi = \frac{(1-\theta)(1-\alpha-\beta)}{1-\theta(1-\upsilon)} l^{\beta} n\bar{x}^{\alpha}, \tag{7a}$$

$$\bar{x} = \frac{\alpha\mu}{1-\alpha}, \tag{16}$$

$$r = \alpha^2 l^{\beta}\bar{x}^{\alpha-1}, \tag{24}$$

$$\eta\frac{c}{1-l} = w, \tag{22}$$

$$\frac{\dot{c}}{c} = \frac{1}{\sigma}(r - \rho), \tag{23}$$

$$\dot{n}\bar{x} = nl^{\beta}\bar{x}^{\alpha} - \mu\dot{n} - c. \tag{25}$$

Eq. (25) is the final goods market clearing condition, or the aggregate resource constraint of the economy, and can be rewritten as:

$$(\bar{x} + \mu)\frac{\dot{n}}{n} = l^{\beta}\bar{x}^{\alpha} - \frac{c}{n}. \tag{26}$$

On the other hand, the labor market clearing condition is that labor demand equals labor supply. By substituting Eq. (5a) into Eq. (22), we obtain:

$$\frac{c}{n} = \frac{1}{\eta}\left(\beta + \frac{\theta\upsilon(1-\alpha-\beta)}{1-\theta(1-\upsilon)}\right)\bar{x}^\alpha \frac{1-l}{l^{1-\beta}}. \tag{27}$$

Substituting Eq. (27) into Eq. (26) yields:

$$\frac{\dot{n}}{n} = \frac{1}{\bar{x}+\mu}\left\{l^\beta \bar{x}^\alpha - \frac{1}{\eta}\left[\beta + \frac{\theta\upsilon(1-\alpha-\beta)}{1-\theta(1-\upsilon)}\right]\bar{x}^\alpha \frac{1-l}{l^{1-\beta}}\right\}. \tag{28}$$

Eq. (28) is referred to as the technology curve, or the technology growth rate (Rivera-Batiz and Romer, 1991).

3   (In)determinacy

By taking logs and the time derivatives of Eqs. (19) and (27), a simple manipulation yields:

$$\sigma\left[\frac{\dot{n}}{n} - \frac{\dot{l}}{1-l} - (1-\beta)\frac{\dot{l}}{l}\right] + \eta(1-\sigma)\frac{\dot{l}}{1-l} = r - \rho. \tag{29}$$

Assume that $\gamma_n(l) = \dot{n}/n$ and $\gamma_c(l) = \dot{c}/c$, which are the technology curve (the innovation growth rate) and the preference curve (the consumption growth rate), respectively. By solving for $\dot{l}$, we can rewrite Eq. (29) as:

$$\dot{l} = \frac{1-l}{\Lambda(l)}[\gamma_n(l) - \gamma_c(l)], \tag{30}$$

where $\Lambda(l) = (1-\beta)(1-l)l^{-1} + 1 - \eta(1-\sigma)\sigma^{-1} > 0$, which is implied by the strict concavity of the utility function. The dynamic motion of the model is determined by the growth rate of innovation and the growth rate of consumption.

Because $\dot{l}$ in Eq. (30) depends only on l, in order to understand the stability properties of the balanced growth equilibrium, we have to identify the sign of $d\dot{l}/dl$ evaluated at $\hat{l}$. A hat over the variables denotes their stationary values. Differentiating Eq. (30) with respect to l, we obtain:

$$\left.\frac{d\dot{l}}{dl}\right|_{l=\hat{l}} = \frac{1-\hat{l}}{\Lambda(\hat{l})}\left[\frac{d\gamma_n(\hat{l})}{dl} - \frac{d\gamma_c(\hat{l})}{dl}\right]. \tag{31}$$

The sign of Eq. (31) is in general undetermined due to the ambiguous sign of the value in the square brackets. The first term in the square brackets on the right-hand side of Eq. (31) is the slope of the technology curve Eq. (28) along the balanced growth equilibrium. The second term is the slope of the preference curve Eq. (23). Hence, the sign of Eq. (31) depends on the relative slopes of these two curves. If the slope of the technology curve is steeper than the slope of the preference curve, then $d\dot{l}/dl > 0$, and thus the fixed point $\hat{l}$ is a repeller, and the dynamic equilibrium will be locally determinate.[5] By contrast, if the preference curve is steeper than the technology curve, $d\dot{l}/dl < 0$, and thus the fixed point $\hat{l}$ is an attractor, and local indeterminacy will emerge in the economy.

In Figs. 1-4, the graph for Eq. (31) shows where the curves respond to the relative slopes of the technology and preferences. The intersection of these two curves denotes the common growth rate of technology and consumption on the vertical axis and the equilibrium level of labor on the horizontal axis. The technology curve is steeper with respect to the l axis than the preference curve at the fixed point E_0, as illustrated in Figs. 1 and 3, which reflects the local determinacy of the equilibrium. Figs. 2 and 4 illustrate the opposite case, which denotes the local indeterminacy of the equilibrium.[6]

Taken together, we have the following proposition:

Proposition 1. *A balanced growth path is locally indeterminate (determinate), if and only if*

$$\Psi \equiv 1 + \frac{\beta + (1-\beta)\hat{l}^{-1}}{\eta}\left(1 + \frac{\theta\upsilon(1-\alpha-\beta)}{(1-\theta(1-\upsilon))\beta}\right) - \frac{1}{\sigma}\alpha \begin{smallmatrix}<\\>\end{smallmatrix} 0.$$

This proposition identifies a source for the emergence of the local indeterminacy of the equilibrium, that is to say, the relative responsiveness of the balanced growth rate of technology and the growth rate of consumption to variations in employment, which in turn determines the sign of Ψ. If $\Psi < 0$, the balanced growth path is locally indeterminate. If $\Psi > 0$, the balanced growth path is locally determinate. That is to say, the relative responsiveness of the balanced growth rate

---

[5]The dynamic system will have a unique perfect foresight equilibrium path if the number of (positively) unstable roots equals the number of jump variables.

[6]For the relative slopes of the two curves, see Appendix 1.

of technology and the growth rate of consumption to variations in employment, which in turn determines the sign of Ψ. Furthermore, if the balanced growth rate of consumption is more (less) responsive to changes in the level of employment than the growth rate of technology, the equilibrium will display indeterminacy (determinacy).

In the model for Proposition 1, a necessary condition for indeterminacy is an intertemporal elasticity of substitution for consumption that is greater than one ($\sigma < 1$).Moreover, this result is the same as in Shaw *et al*. (2005) who find a determinate balanced growth equilibrium if the intertemporal elasticity of substitution is smaller than 1, and in Pelloni and Waldmann (2000) and Haruyama and Itaya (2006), who find that a necessary condition for local indeterminacy is one where the intertemporal substitution of consumption is elastic (i.e., $\sigma < 1$). Hence, we have Proposition 2:

Proposition 2. *Following Proposition 1, when the elasticity of substitution σ^{-1} is greater than 1, then the balanced growth equilibrium may be locally indeterminate. If the elasticity of substitution σ^{-1} is equal to or less than 1, then the balanced growth equilibrium will be locally determinate.*[7]

According to the dynamics of the model, suppose that the initial level of l is lower than the balanced growth equilibrium level of $\hat{l}$, which makes the production of final goods lower. On the other hand, a lower l causes the consumption to rise since the consumption and leisure are complementary goods. These two effects together mean that fewer resources are devoted to R&D, and describe the economy that is located to the south west of the initial equilibrium E_0 in Figs. 1-4. Starting from a southwest point of E_0 along the technology curve, whether the economy converges to or diverges from the balanced growth equilibrium E_0 will depend on whether $\Psi < 0$ holds or not, namely, on the relative magnitude of the responses of γ_n and γ_c to changes in l (i.e., Eq. (31)).

If the technology curve is very sensitive to changes in l compared to the preference curve, the technology curve will be steeper than the preference curve (i.e.,

---

[7]Because the requirement for the utility function is concave, or $\sigma > \eta/(1 + \eta)$, we can easily derive $\sigma < 1$, which is the necessary condition for local indeterminacy. Also see Appendix 2.

$\dot{l}/l > 0$) as illustrated in Figs. 1 and 3. A point southwest of E_0 indicates that the growth rate of consumption is greater than that of technology, and that the level of l is lower than $\hat{l}$. The lower initial level l means that in the next period labor will decline because of $\dot{l}/l > 0$. This fall in l causes the economy to move away from the balanced growth equilibrium, the economy is unstable and thus the l should jump to the balanced growth equilibrium level. The balanced growth equilibrium is determinate.

By contrast, if the preference curve is very sensitive to changes in l compared to the technology curve, the preference curve will be steeper than the technology curve (i.e., $\dot{l}/l < 0$), as illustrated in Figs. 2 and 4. A point south west of E_0 indicates that the growth rate of technology is higher than that of consumption, and the level of l is lower than $\hat{l}$. The lower initial level of l means that in the next period labor will increase because of $\dot{l}/l < 0$. As a result, l and γ_n can gradually return to the balanced growth equilibrium along the technology curve, and the balanced growth equilibrium will be stable, that is, indeterminate.
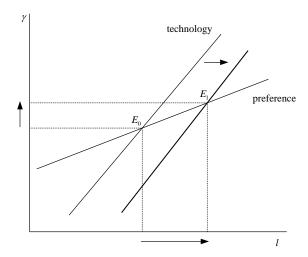

Figure 1. The Effects of an Increase in the Union's Bargaining Power θ (or the Extent to which the Union is Employment-oriented υ) in a Determinate Equilibrium
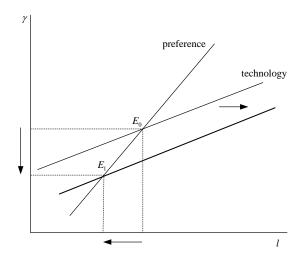
Figure 2. The Effects of an Increase in the Union's Bargaining Power θ (or the Extent to which the Union is Employment-oriented υ) in an Indeterminate Equilibrium
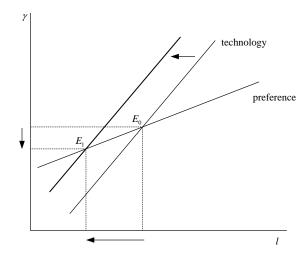

Figure 3. The Effects of an Increase in the Extent to which the Union is Wage-oriented $1-\upsilon$ in a Determinate Equilibrium
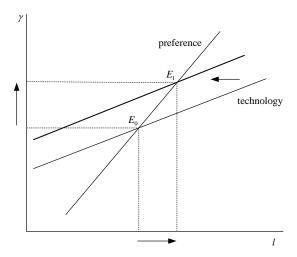
Figure 4. The Effects of an Increase in the Extent to which the Union is Wage-oriented $1-\upsilon$ **in**

an Indeterminate Equilibrium

4 Growth Effects of Unionization

We investigate the long-run impact of unionization (union bargaining power θ, the extent to which the union is employment-oriented υ, and the extent to which the union is wage-oriented $1-\upsilon$) on the balanced growth path of the present model. We examine the effects of changes in the union's bargaining power, the extent to which the union is employment-oriented, and the extent to which the union is wage-oriented on the level of employment and thus on the growth rate along the balanced growth path.

In the steady state, $\dot{l}=0$ in Eq. (30). Totally differentiating Eq. (30) with respect to θ, υ yields:

$$\frac{d\hat{l}}{d\theta} = \frac{1}{\Psi} \frac{(1-l)}{\eta} \frac{1-\alpha-\beta}{\beta} \frac{\upsilon}{(1-\theta+\theta\upsilon)^2} , \tag{32}$$

$$\frac{d\hat{l}}{d\upsilon} = \frac{1}{\Psi} \frac{(1-l)}{\eta} \frac{1-\alpha-\beta}{\beta} \frac{\theta(1-\theta)}{(1-\theta+\theta\upsilon)^2} , \tag{33}$$

which reveals that the effects of a change in the union's bargaining power, and the

extent to which the union is employment-oriented are governed solely by the sign of Ψ; that is, on whether the balanced growth path displays local determinacy or indeterminacy. In fact, a higher degree of union bargaining power or a higher extent to which the union is employment-oriented or a lower extent to which the union is wage-oriented raises (reduces) the level of employment, if and only if the balanced growth path displays local determinacy (indeterminacy).

Differentiating Eq. (28) with respect to θ, we obtain:

$$
\frac{d\gamma_n}{d\theta} = \frac{\beta \hat{l}^{\beta-1} \bar{x}^{\alpha}}{\bar{x} + \mu} \left[1 + \frac{\beta + (1-\beta)\hat{l}^{-1}}{\eta} \left(1 + \frac{\theta \upsilon (1-\alpha-\beta)}{(1-\theta(1-\upsilon))\beta} \right) \right] \frac{d\hat{l}}{d\theta}. \tag{34}
$$

This amounts to:

$$
\text{sign}(\frac{d\gamma_n}{d\theta}) = \text{sign}(\frac{d\hat{l}}{d\theta}) = \text{sign}(\Psi). \tag{35}
$$

This positive association between the growth rate and the level of employment is apparently consistent with Figs. 1 and 2. In addition to the union's bargaining power, we also have the same result for the extent to which the union is employment-oriented υ. However, according to the extent to which the union is wage-oriented $1-\upsilon$, we have the opposite effect when compared with the effect of the union's bargaining power on the growth rate and level of employment which is consistent with Figs. 3 and 4.

Proposition 3. *An increase in the union's bargaining power raises (reduces) the equilibrium level of employment as well as the balanced growth rate, if and only if the balanced growth equilibrium is locally determinate (indeterminate). Furthermore, an increase in the degree of employment-oriented raises (reduces) the equilibrium level of employment as well as the balanced growth rate, if and only if the balanced growth equilibrium is locally determinate (indeterminate). Conversely, an increase in the extent to which the union is wage-oriented reduces (raises) the equilibrium level of employment and the balanced growth rate, if and only if the balanced growth equilibrium is locally determinate (indeterminate).*

Suppose that the economy is initially at the balanced growth equilibrium E_0

in Figs. 1-4. At first, we consider an unanticipated and permanent increase in the union's bargaining power (or the case where the union is employment-oriented) and this shifts the technology curve downward.[8] However, the resulting effects on the level of employment and the balanced growth rate depend on the stability characteristics of the equilibrium, namely, the relative slopes of the technology and preference curves.

In Fig. 1, the technology curve is steeper than the preference curve, thereby depicting a determinate equilibrium case. The new equilibrium E_1 features larger values of both l and γ that are generated when the technology curve shifts downward. At the old equilibrium E_0, the steady state growth rate of consumption is larger than that of technology, and thus the ratio c/n begins to increase, the employment falls which in turn further drives the economy away from the new balanced growth equilibrium. As a result, the economy must jump to the new balanced growth equilibrium. Hence, the economy is unstable and thus the balanced growth equilibrium is determinate. To provide intuition for this result, an increase in union bargaining power (or where the union is employment-oriented) will increase the demand for labor and thus increase the employment level. This will increase the final goods produced. Thus, the more resources that are devoted to R&D, the higher the economic growth rate that will result.

In the case of an indeterminate balanced growth equilibrium, the preference curve is steeper than the technology curve, as shown in Fig 2. Increasing the union's bargaining power (or the extent to which the union is employment-oriented) will cause the technology curve to shift downward. The new equilibrium E_1 featuring a smaller l and a smaller γ will be generated. Because the ratio c/n increases, employment decreases. This movement in l induces the economy to gradually approach the new balanced growth equilibrium. An increase in union bargaining power (or the extent to which the union is employment-oriented) leads to a decrease in the final goods firm's bargaining power, which will decrease the profit obtained by the final goods firm. This will reduce the employment of labor in production, and thus reduce the final goods produced. Hence, the fewer the resources that are devoted to R&D activities, the less economic growth that will be generated.

To consider the growth effect of the union's membership power, which is the

---

[8]See Eqs. (A15) and (A16) in Appendix 3.

extent to which the union is wage-oriented, we first examine the case of a determinate balanced growth equilibrium. In Fig. 3, an increase in the extent to which the union is wage-oriented will shift the technology curve upward,[9] and the new equilibrium E_1 featuring a small l and a small γ will be obtained. At the old equilibrium point E_0 the growth rate of consumption is less than that of technology, thus leading to a smaller c/n. Therefore, employment will begin to rise, which in turn will drive the economy further away from the new balanced growth equilibrium. As a result, both l and γ should instantaneously jump to the new balanced growth equilibrium. That is, if the union's membership that prefers a higher labor wage has dominant power, this will cause the final goods firm's costs to increase. Therefore, the final goods firm will employ less labor, and thus employment will decline. On the other hand, the fewer the intermediate goods that are used in the final goods market, the less R&D activity there will be, and thus less economic growth will be generated.

Conversely, in an indeterminate case, the new balanced growth equilibrium E_1 will be located to the northeast of the original intersection point E_0 when the union is wage-oriented, as illustrated in Fig. 4. The figure shows that the union's membership will boost economic growth and employment at point E_1. Because the ratio c/n declines, employment is increased. This movement in l allows the economy to gradually approach the new balanced growth equilibrium. If the union is wage-oriented, the higher wage will increase the labor supply, and will thus increase employment. This will cause the final goods firm to increase the intermediate goods used, which will then increase the R&D activity. Thus the economic growth will be boosted.

Figs. 1 and 4 contribute to an important macro implication, that is, unionization will not necessarily be bad for employment and growth if the union is employment-oriented in a determinate equilibrium and is wage-oriented in an indeterminate equilibrium. This finding is as the same as those of Palokangas (1996), who provides an example to propound the possibly positive relationship between unionization and economic growth in an R&D growth model with two labor sectors (the skilled and unskilled sectors), Irmen and Wigger (2000), who develop an OLG model with a trade union and find that the unionization may lead to higher aggregate

---

[9]See Eq. (A17) in Appendix 3.

savings and per capita income growth, and Chang *et al*. (2007), who induces the internal conflict within a political union and indicates that a higher degree of unionization will result in a lower unemployment rate and a higher balanced growth rate if the union is employment-oriented in an AK growth model.

Figs. 2 and 3 provide that the unionization is not good for employment and growth if the union is employment-oriented in an indeterminate equilibrium and is wage-oriented in a determinate equilibrium. These results are the same as in Lingens (2007), who finds that unions would give rise to a hold-up problem which would decrease the incentive to invest in research, with the result that the rate of growth would decline, and Chang *et al*. (2007), who point out that unionization will harm both employment and growth if the union is wage-oriented.

In a way that differs from their respective approaches, we abstract the interaction between sectors and the allocation of resources between sectors from our analysis and show that the endogenous labor supply can lead to fundamental changes in the stability of the long-run equilibrium. This is because the determinant of the Jacobian matrix of the dynamic system, evaluated at a steady state, will exhibit the opposite sign depending on whether the equilibrium is determinate or indeterminate. If indeterminacy occurs, the steady state comparative statics properties may be reversed. Hence, we provide a general outcome when the equilibrium is determinate or indeterminate and can explain the overall results for the relevant literature.

We follow the practice of many articles which analyze the comparative statics when the steady state equilibrium is indeterminate. Such as Itaya and Mino (2003), Haruyama and Itaya (2006), Itaya (2008), Itaya and Kanamori (2010), they analyze the comparative statics when the steady sate equilibrium is indeterminate.

However, growth-enhancing union effects arise for reasons not related to the issue of whether equilibria are determinate or indeterminate. As we have already seen, the present paper offers an alternative theory that points toward growth-enhancing effects of union on the basis of the stability nature of long-run equilibrium in an R&D-based endogenous growth model.

5   Conclusion

In this paper, we incorporate a labor union into an R&D endogenous growth model with elastic labor. We show that the stability properties of the long-run equilibrium play a decisive role in the relationship between unionization, employment, and economic growth. Regardless of whether the balanced growth equilibrium is stable (indeterminate) or unstable (determinate), the influence on unionization will ambiguously improve economic growth and employment. If the households' intertemporal elasticity of substitution is greater (less) than 1, then the balanced growth equilibrium may be locally indeterminate (determinate). It may thus be concluded that when a balanced growth path displays indeterminacy, an increase in the extent to which the union is wage-oriented has a positive impact on long-run growth and employment. Furthermore, when a balanced growth path exhibits determinacy, a high degree of union bargaining power or a union that is employment-oriented will result in higher employment and a higher balanced growth rate.

Since this paper assumes that household endogenously chooses the labor supply, it is limited in this model. However, some studies show that the household can endogenously choose the work hours, even if the union and employer have bargained over the employment, such as Chang and Hung (2016). This paper introduces endogenous labor supply which labor will grow. We need to drive the labor supply for solve the wage. If we change endogenous labor supply to endogenous work hours, the endogenous labor hours will go to a certain value. This is the different between endogenous labor supply and endogenous labor work hours. It is very interesting to study the choice of consumer work hours, so we will study this part in the future.

Appendix 1

Substituting Eq. (24) into Eq. (23), yields the optimal rate of growth of consumption as follows:

$$\gamma_c(\hat{l}) = \left.\frac{\dot{c}}{c}\right|_{l=\hat{l}} = \frac{1}{\sigma}\left(\alpha^2 \hat{l}^\beta \bar{x}^{\alpha-1} - \rho\right). \tag{A1}$$

By differentiating the right-hand side of Eq. (A1) with respect to $\hat{l}$ we obtain:

$$\frac{d\gamma_c}{dl} = \frac{1}{\sigma}\beta\alpha^2\hat{l}^{\beta-1}\bar{x}^{\alpha-1} > 0, \tag{A2}$$

which implies that the slope of the preference curve is positive.

As to the technology curve, differentiating Eq. (28) with respect to $\hat{l}$ leads to:

$$\left.\frac{d\gamma_n}{dl}\right|_{l=\hat{l}} = \frac{\hat{l}^{\beta-1}\bar{x}^{\alpha}}{(\bar{x}+\mu)}\left[\beta + \frac{1+(1-\beta)(1-\hat{l})\hat{l}^{-1}}{\eta}\left(\beta + \frac{\theta\upsilon(1-\alpha-\beta)}{1-\theta(1-\upsilon)}\right)\right] > 0, \tag{A3}$$

which implies that the slope of the technology curve is also positive.

Combining Eq. (A3) with Eq. (A2), and using $(\bar{x}+\mu)^{-1} = (1-\alpha)/\mu$ gives:

$$\frac{d\gamma_n}{dl} - \frac{d\gamma_c}{dl} = \frac{(1-\alpha)\beta\hat{l}^{\beta-1}\bar{x}^{\alpha}}{\mu}\left\{1 + \frac{\beta+(1-\beta)\hat{l}^{-1}}{\eta}\left(1 + \frac{\theta\upsilon(1-\alpha-\beta)}{(1-\theta(1-\upsilon))\beta}\right) - \frac{1}{\sigma}\alpha\right\}. \tag{A4}$$

Eq. (A4) reveals the sign of Eq. (31). By assuming that the brace on the right-hand side of Eq. (A4) is:

$$\Psi \equiv 1 + \frac{\beta+(1-\beta)\hat{l}^{-1}}{\eta}\left(1 + \frac{\theta\upsilon(1-\alpha-\beta)}{(1-\theta(1-\upsilon))\beta}\right) - \frac{1}{\sigma}\alpha \underset{<}{\overset{>}{}} 0, \tag{A5}$$

we then have Proposition 1.

Appendix 2

From Eq. (A5), the economy exist indeterminate equilibrium if and only if $\Psi < 0$.

Rearrange Eq. (A5):

$$1 - \frac{1}{\sigma}\alpha < -\frac{\beta+(1-\beta)\hat{l}^{-1}}{\eta}\left(1 + \frac{\theta\upsilon(1-\alpha-\beta)}{(1-\theta(1-\upsilon))\beta}\right). \tag{A6}$$

Since the right hand side of the equation is negative, the left hand side of the equation must be negative too. Note first that when $\sigma > 1$, the left hand side of (A6)

cannot be negative. Hence we have $\sigma < 1$ which is a necessary condition for indeterminacy.

In addition, we can also provid $\sigma < 1$. Because the requirement for the utility function is concave, we can easily derive $\sigma < 1$, which is the necessary condition for local indeterminacy, as follows

From household's utility function:

$$V(c, l) = \frac{(c(1-l)^{\eta})^{1-\sigma}}{1-\sigma}. \tag{A7}$$

Make first and second order differentials for c and l respectively:

$$V_c = c^{-\sigma}(1-l)^{\eta-\eta\sigma} > 0, \tag{A8}$$
$$V_l = -\eta c^{1-\sigma}(1-l)^{\eta-\eta\sigma-1} < 0, \tag{A9}$$
$$V_{cc} = -\sigma c^{-\sigma-1}(1-l)^{\eta-\eta\sigma} < 0, \tag{A10}$$
$$V_{ll} = \eta(\eta-\eta\sigma-1)c^{1-\sigma}(1-l)^{\eta-\eta\sigma-2} > 0, \tag{A11}$$
$$V_{cl} = V_{lc} = -(\eta-\eta\sigma)c^{-\sigma}(1-l)^{\eta-\eta\sigma-1} < 0, \tag{A12}$$

Since the utility function is concave in c and $1-l$, we have second order condition as follows:

$$V_{cc} < 0, \tag{A13}$$
$$\begin{vmatrix} V_{cc} & V_{cl} \\ V_{lc} & V_{ll} \end{vmatrix} > 0 \quad \text{if} \quad \sigma > \frac{\eta}{1+\eta}. \tag{A14}$$

We can easily derive $\sigma < 1$.

Appendix 3

By partially differentiating Eq. (28) with respect to θ, υ, and $1-\upsilon$, we obtain:

$$\frac{\partial \gamma_n}{\partial \theta} = -\hat{l}^{\beta-1}\bar{x}^{\alpha}(1-\alpha-\beta)\frac{(1-\alpha)}{\mu}\frac{(1-l)}{\eta}\frac{\upsilon}{(1-\theta+\theta\upsilon)^2} < 0, \tag{A15}$$

$$\frac{\partial \gamma_n}{\partial \upsilon} = -l_t^{\beta-1}\bar{x}^{\alpha}(1-\alpha-\beta)\frac{1-\alpha}{\mu}\frac{1-l}{\eta}\frac{\theta\upsilon}{1-\theta+\theta\upsilon}\frac{\theta(1-\theta)}{(1-\theta+\theta\upsilon)^2} < 0, \tag{A16}$$

$$\frac{\partial \gamma_n}{\partial (1-\upsilon)} = l_t^{\beta-1}\bar{x}^{\alpha}(1-\alpha-\beta)\frac{1-\alpha}{\mu}\frac{1-l}{\eta}\frac{\theta\theta\upsilon}{[1-\theta(1-\upsilon)]^2} > 0, \tag{A17}$$

References

Aghion, P. and P. Howitt, (1992), "A Model of Growth through Creative Destruction," *Econometrica*, 60, 323-351.

Aghion, P. and P. Howitt, (2009), *Ecndogenous Growth Theory*, Cambridge, MA, MIT Press.

Benhabib, J. and R. Farmer, (1994), "Indeterminacy and Increasing Returns," *Journal of Economic Theory*, 63, 19-41.

Boone, J., (2000), "Technological Progress, Downsizing and Unemployment," *Economic Journal*, 110, 581-600.

Booth, A. L., (1995), *The Economics of the Trade Union*, Cambridge; New York: Cambridge University Press.

Bräuninger, M., (2000), "Wage Bargaining, Unemployment, and Growth," *Journal of Institutional and Theoretical Economics*, 156, 646-660.

Chang, J. J. and H. W. Hung, (2016), "Trade Unions, Unemployment, Economic Growth, and Income Inequality," *Macroeconomic Dynamics*, 20, 404-428.

Chang, J. J., M. F. Shaw, and C. C. Lai, (2007), "A "Managerial" Trade Union and Economic Growth," *European Economic Review*, 51, 365-384.

Chen, P. H., A. C. Chu, H. Chu, and C. C. Lai, (2017), "Short-Run and Long-Run Effects of Capital Taxation on Innovation and Economic Growth," *Journal of Macroeconomics*, 53, 207-221.

Clark, A., (1990), "Efficient Bargains and the McDonald-Solow Conjecture," *Journal of Labor Economics*, 8, 502-528.

De Groot, H. L. F., (2001), "Unemployment, Growth, and Trade Unions," *Growth and Change*, 32, 69-91.

Faini, R., (1999), "Trade Unions and Regional Development," *European Economic Review*, 43, 457-474.

Grossman, G. M. and E. Helpman, (1991), *Innovation and Growth in the Global Economy*, Cambridge, MA: MIT Press.

Haruyama, T. and J. I. Itaya, (2006), "Do Distortionary Taxes Always Harm Growth," *Journal of Economics*, 87, 99-126.

Haucap, J. and C. Wey, (2004), "Unionisation Structures and Innovation Incentives," *Economic Journal*, 114, C149-C165.

Irmen, A. and B. U. Wigger, (2000), "Trade Union Objectives and Economic Growth," Unpublished Manuscript, University of Mannheim.

Itaya, J.-I., (2008), "Can Environmental Taxation Stimulate Growth? The Role of Indeterminacy in Endogenous Growth Models with Environmental Externalities," *Journal of Economic Dynamics & Control*, 32, 1156-1180.

Itaya, J.-I. and K. Mino, (2003), "Inflation, Transaction Costs and Indeterminacy in Monetary Economies with Endogenous Growth," *Economica*, 70, 451-470.

Itaya, J.-I. and N. Kanamori, (2010), "Consumption Taxation, Social Status and Indeterminacy in Models of Endogenous Growth with Elastic Labor Supply," *Journal of Economics*, 100, 141-163.

Layard, R., S. Nickell, and R. Jackman, (1991), *Unemployment: Macroeconomic Performance and the Labour Market*, New York: Oxford University Press.

Lingens, J., (2003), "The Impact of a Unionised Labour Market in a Schumpeterian Growth Model," *Labour Economics*, 10, 91-104.

Lingens, J., (2007), "Unions, Wage Setting, and Economic Growth," *Economic Modelling*, 24, 167-188.

Lingens, J., (2009), "Union Wage Bargaining and the Incentive to Innovate," *Journal of Institutional and Theoretical Economics*, 165, 260-280.

McDonald, I. M. and R. M. Solow, (1981), "Wage Bargaining and Employment," *American Economic Review*, 71, 896-908.

Menezes-Filho, N. and J. Van Reenen, (2003), "Unions and Innovation: A Survey of the Theory and Empirical Evidence," *The International Handbook of Trade Unions*, 293-335, Cheltenham, U.K. and Northampton, Mass.: Elgar.

Palokangas, T., (1996), "Endogenous Growth and Collective Bargaining," *Journal of Economic Dynamics and Control*, 20, 925-944.

Palokangas, T., (2004), "Union-Firm Bargaining, Productivity Improvement and Endogenous Growth," *Labour*, 18, 191-205.

Pelloni, A. and R. Waldmann, (2000), "Can Waste Improve Welfare?" *Journal of Public Economics*, 77, 45-79.

Rivera-Batiz, L. A. and P. M. Romer, (1991), "Economic Integration and Endogenous Growth," *Quarterly Journal of Economics*, 106, 531-555.

Romer, P. M., (1990), "Endogenous Technological Change," *Journal of Political Economy*, 98, 71-102.

Shaw, M. F., S. H. Chen, and J. J. Chang, (2005), "Trade Union, Fiscal Policy and Economic Growth: An Endogenous Growth Model with Equilibrium Unemployment," *Academia Economic Papers*, 33, 323-350.

Ulph, A. M. and D. T. Ulph, (1994), "Labour Markets and Innovation: Ex-Post Bargaining," *European Economic Review*, 38, 195-210.