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Taiwan Stock Market and the Macroeconomy: A Smooth-Transition Approach

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We employ a two-regime smooth transition regression model with a logistic transition function to measure the degree of the nonlinear interaction between the Taiwanese stock market and macroeconomic indexes for economic growth (GDP), price stability (CPI), money growth (M2), risk-free rate (Taiwan T-Bills Rate), US exchange rate, and the US stock market's Dow Jones index. Starting with eight lags for each variable, we employ the LASSO statistical methodology to pick a few significant regressors, and building on the resulting specification, we find strong statistical evidence of nonlinearity, with the Dow Jones index playing the switching variable. The results of the fitted smooth transition model suggest two distinct bull and bear-type regimes for the stock market index with complex, significant, and asymmetric effects due to its lags and other variables. In a simple 4- and 8-step ahead forecasting exercise, our nonlinear model does not seem to outperform the linear specification for most forecasting accuracy measures employed; hence, we are not able to confirm recent results by Guidolin *et al.* (2014), suggesting that nonlinear models forecast better. Finally, we fit linear and nonlinear specifications by splitting our dataset into two periods, the pre- and post-Great Financial Crisis, as dated by the NBER, and we find intricate nonlinear effects differing between the two periods with the post-GFC fit being better in all statistical measures employed.

Keywords: Nonlinear modeling, smooth transition, LASSO, forecasting.

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JEL classification: C01, C52.

1 Introduction

Regime-switching models provide a flexible way to model nonlinearities often found in data. In recent years, there has been an increased interest in applying such models to finance, economics, and empirical macroeconomics, in particular, due to the interest in capturing phenomena associated with the business cycle. By allowing for distinct states (or regimes) of the world, these nonlinear models can represent situations where mean behavior depends on the regime, with a positive mean during business cycle expansions and a negative mean during recessions.

Regime-switching models differ in the way the regime evolves over time, and two main classes of models can be distinguished. In one class, we assume that the regimes can be characterized by an observable variable and hence the regimes that have occurred in the past and present are known with certainty. In another class, we assume the regime cannot actually be observed but is determined by an underlying unobservable stochastic process. This implies that one can never be certain that a particular regime has occurred at a particular point in time but can only assign probabilities to the occurrence of the different regimes that, in turn, need to be estimated.

A way of modelling the switching that happens between economic regimes is by Markov switching-type models originally due to Hamilton (1989) in which such switching is governed by a regime-dependent probability. Another way is to apply threshold regression-type models, pioneered by Tong (1978, 1990), where the regime is determined by the (threshold) value of some observable variable called the threshold variable, which has to be identified and its threshold estimated. Such models have been used extensively in econometrics to estimate mean effects, Hansen (1996, 2000, 2011), Enders et al. (2007), or quantile effects, Caner (2002), Galvao et al. (2011, 2014), and Kuan et al. (2017) among others.

Another flexible way to model the cyclical behavior of some variables of interest is to use a smooth transition (auto) regression model (STAR) pioneered by Terasvirta (1988). In such models, the switching happens smoothly, controlled by a distribution function (logistic or exponential) together with a parameter that determines the magnitude of such switching. Although the idea of a smooth transition between regimes goes back as far as 1971 due to Bacon and Watts, their introduction into the nonlinear time-series literature occurred with Chan and Tong (1986) and was further popularized by Granger and Terasvirta (1993) and Terasvirta (1994). For a comprehensive review of the STAR model and extensions that allow for exogenous variables as regressors, see Terasvirta (1998), while for applications in economics and finance, see Cao and Tsay (1992), Domian and Louton (1997), and Peel and Speight (1998) among others. Van Dijk, Terasvirta, and Franses (2002) provide a nice overview.

Our aim is to model the cyclical behavior of the Taiwanese stock exchange market as captured by its index called TAIEX, using key macroeconomic variables and a stock market index from the United States, by considering a smooth-transition model. Studies on regime-switching models applied to stock price movements are not new and include Michael, Nobay and Peel (1997), Perez-Quiros and Timmermann (2000), Sarantis (2001), Ang and Bekaert (2002), and Guidolin and Timmermann (2003). For the nonlinear interaction of the stock market and the macroeconomy, see Fama (1981, 1990), Schwert (1990), Flannery and Propopapakis (2002), and Guidolin and Ono (2007) among others. It is also well known that non-linearity exists in the volatility equation when dealing with a financial time series, and the threshold smooth-transition model has been successfully employed for this purpose in Gerlach and Chen (2008), Lin *et al.* (2012), and Chen *et al.* (2017) among others. However, we do not focus on asymmetric volatility effects in this paper.

Taiwan is an economically developed country, also known as an Asian tiger, whose economic policies have markedly increased its citizens' prosperity and welfare over the last 40 years. It is advisable to use the smooth-transition model approach since we expect a large number of investors to react at different times to different economic signals, hence leading the financial index to a gradual switching from one regime to another. In our analysis, we employ macroeconomic indexes for economic growth (GDP), price stability (CPI), money growth (M2), a proxy for the risk free rate (Taiwan T-Bills Rate), the exchange rate USD/NTD (New Taiwan Dollar), and a stock market index from the US (the Dow Jones) by fitting a two-state smooth transition model. We first select the best linear model that fits the data well enough using different statistical selection criteria. By employing Granger's (1993) strategy in fitting nonlinear models to data, we start by finding the best linear specification, using eight lags for each variable. Due to a large number of potential explanatory variables, we resort to statistical techniques of variable selection (stepwise regression and LASSO) to pick a sufficient number of regressors that explain the TAIEX data well enough but also avoiding overfitting the data. Based on the selected model, we conduct nonlinearity tests and simultaneously find strong statistical evidence of nonlinearities, with the Dow Jones index playing the role of the switching variable. The results of the fitted, smooth transition model suggest two distinct bull and bear-type regimes for the Taiwan stock market index with complex, significant, and asymmetric effects due to its lags as well as the GDP, CPI, USD/NTD, M2, and lagged Dow Jones returns. In addition, diagnostic tests validate the chosen nonlinear specification. In a simple forecasting exercise, our nonlinear model does not perform that well compared to the simple linear specification for most forecasting evaluation measures employed, so more work is needed in this area to see if these results are robust. Finally, we employ additional analyses by fitting linear and nonlinear specifications by splitting our dataset into two periods, the pre- and post-Great Financial Crisis dated at the third quarter of 2008 by the NBER. The nonlinear effects are different between the two periods with the post-GFC fit being better in all statistical measures employed.

The paper is organized a follows: in the next section we review the methodology of smooth-transition (auto) regressive models and fix the notation. Section 3 briefly presents statistics and describes the data set used in our analysis while section 4 specifies and fits a linear model on that data set. Section 5 contains the results for fitting a smooth transition (auto) regressive model and interpretations. Section 6 is devoted to a forecasting exercise while section 7 splits the sample into two periods based on the occurrence of the Great Financial Crisis and re-estimates the model. Section 8 concludes this work, and the Appendix contains additional plots of our data.

2 Methodology

A smooth-transition (auto) regressive model (ST(A)R) has the following general representation:

$$y_t = \emptyset_1^T y_{t,p}[F(Z_{t-d};\gamma,\alpha,q)] + \emptyset_2^T y_{t,p}[1 - F(Z_{t-d};\gamma,\alpha,q)] + \epsilon_t \tag{1}$$

where $y_{t,p}$ is a vector that contains lags up to order p of the stationary dependent variable y_t, and 1 for the intercept term. That is $y_{t,p} = (1, y_{t-1}, \dots, y_{t-p})^T$, $\emptyset_i = (\emptyset_{i0}, \emptyset_{i1}, \dots, \emptyset_{ip})^T$ is the vector of parameters with i=1,2, assuming only two regimes, $\alpha = (\alpha_1, \dots, \alpha_k)^T$,[1] z_{t-d} is the regime(state)-switching variable that can be a vector of switching variables: $Z_{t-d} = (Z_{1,t-d,} \dots, Z_{j,t-d})^T$, j=1,⋯,k, is a vector of k observed variables assumed to explain the regime transition with lag (delay) parameter(s) d, while ϵ_i is a non-identically, independently distributed random variable with mean zero and variance σ^2 >0. To complete the description of the model, we discuss the regime- transition function $F(\cdot)$, a continuous function bounded on the unit interval, which comes in two types:

$$\text{Logistic: } F(Z_{t-d}; \gamma, \alpha, q) = \frac{1}{1+\exp(-\gamma(\alpha^T z_{t-d} - q))}, \gamma > 0 \tag{2}$$

$$\text{Exponential: } F(Z_{t-d}; \gamma, \alpha, q) = (1 - \exp(-\gamma(\alpha^T Z_{t-d} - q)^2)), \gamma > 0 \tag{3}$$

In the above, q is the change-point (threshold value), and γ is a parameter that determines the smoothness (or speed) of the transition from one regime to another. The following plot of the two functions above displays their sensitivity to the parameter γ, where, in the case of the logistic function, we get the non-smooth threshold autoregressive model (TAR), as γ → ∞.

[1] More regimes can also be considered but at the expense of interpretability.

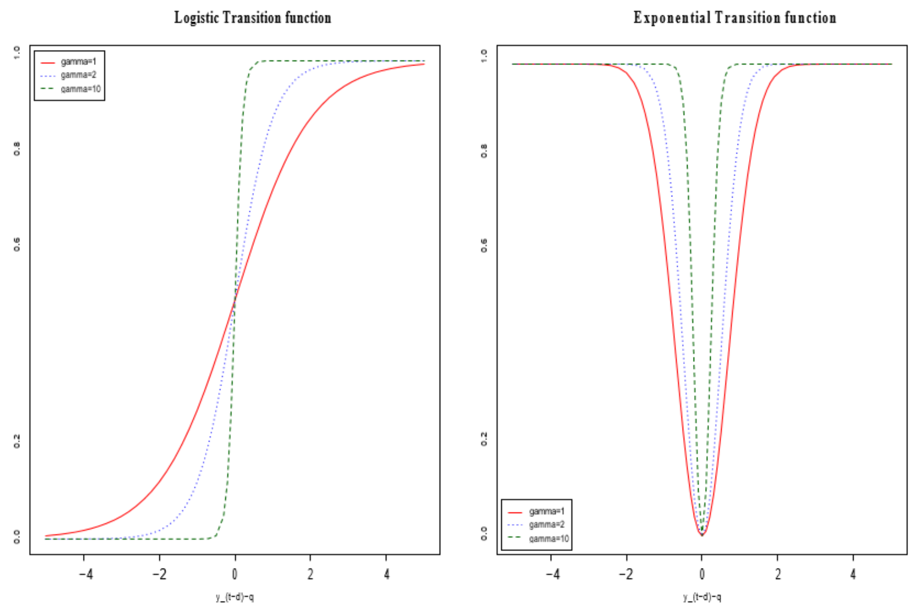

Figure 1. Transition Functions

Although many papers, Aslanidis *et al*. (2002) among others, try to identify which specification, the logistic or exponential, is the best for a data set, Here, we only try the logistic specification as our transition function. Recently, Buncic (2018) showed that using the exponential function as a transition function can be ill-suited due to the fact that (i) when the transition parameter γ is small, it can be approximated by a quadratic function, causing identification issues and (ii) when the transition parameter γ is large, the exponential function behaves like an indicator function, spuriously overfitting a small number of observations around the location parameter. Both issues can lead to serious estimation problems, as Buncic (2018) shows both analytically and numerically; hence, our preference for the logistic specification.

Estimation of the ST(A)R model is done by maximizing the likelihood, and it can be quite complicated in some cases, especially when the errors follow some time-varying volatility model like GARCH; see Chan and McAleer (2002, 2003). A danger in fitting a highly nonlinear model is data overfitting, usually seen through an extremely high R^2 statistic, especially when many regressors are used as explanatory variables. Therefore, after discussing the data employed in our analysis, we proceed to select carefully, using modern statistical techniques, the appropriate number of regressors to be used when fitting the nonlinear model so as to avoid the data overfitting problem.

3 Data Description

We have collected quarterly Taiwan economy data from 1984(Q4) to 2017(Q4) from the Yahoo Finance website[2]. The CPI base year changes and re-calibrates every five years, so the base year used is 2016 while both GDP and CPI are seasonally adjusted. Real GDP data is calculated by deflating the nominal GDP data with the Taiwan deflator[3]. Data for the risk-free rate, called TWRATE here (MB64), is converted from monthly to quarterly values by the geo-mean method and obtained from the Central Bank of the Republic of China (Taiwan)[4]. The Taiwan/US dollar exchange rate dataset is obtained from the FRED data base[5] while data for the Money supply M2 is obtained from Datastream. All of the stock market indexes are adjusted closing prices of the last trade day. The following plot is the nominal versus real GDP data for Taiwan for the period 1984(Q4) to 2017(Q4).

[2] https://finance.yahoo.com/
[3] https://www.ceicdata.com/en/taiwan/sna-08-reference-year2011-gdp-deflator-2011-price/gdp-deflator-yoy
[4] Obtained from the TEJ data base: http://www.finasia.biz/ensite/.
[5] https://fred.stlouisfed.org

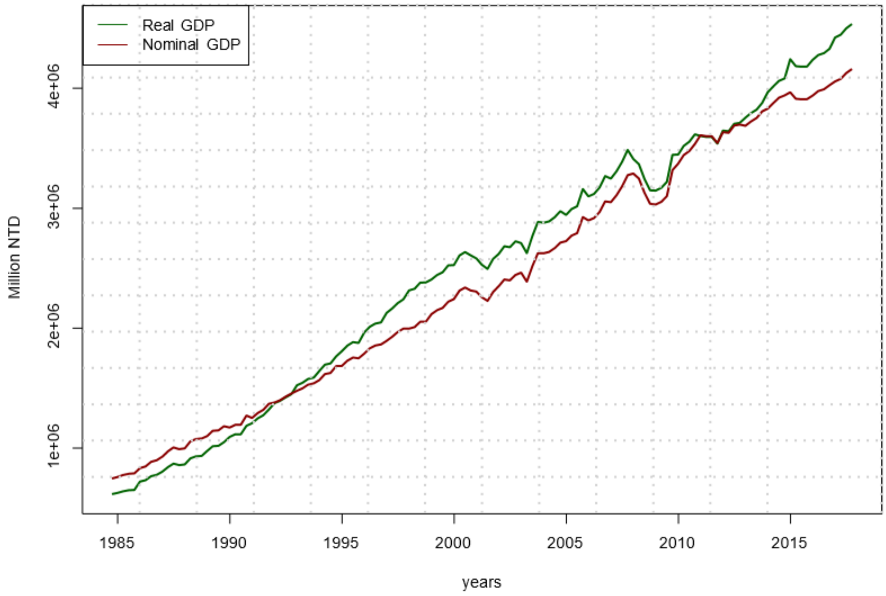

Figure 2. Real vs Nominal Taiwan GDP (2011=Base year)

Following Aslanidis (2002), we believe that quarterly data best reflects the effect of the macroeconomic variables on the stock market. In more detail, the stock market in Taiwan is represented by TAIEX, the main stock market index, while Taiwan's macroeconomy is represented by the following variables: Gross Domestic Product for economic activity, Consumer Price Index for inflation, M2 as a money supply variable and a proxy for the risk free rate, and the Taiwan central bank T-Bills 1-30 days market rate (MB64). We have also used the Taiwan/US dollar exchange rate since Taiwan's economy is strongly export-oriented, and its economic performance depends heavily on its currency being "cheap" enough relative to the dollar to have a price advantage in product valuation. For the US stock market, we have three stock market indexes, the Dow Jones, the SP500, and the NASDAQ. Plots and histograms of each variable are displayed in the Appendix. Here, we only give a basic statistical description of our data.

Table 1. Basic Statistics

Variables	Mean	Min	Max	St.Dev
TAIEX	0.01925	-0.75615	0.99481	0.20911
Taiwan GDP	0.015212	-0.037348	0.102339	0.020393
Taiwan T-Bills Rate	0.0324	0.0035	0.0883	0.02296
Taiwan CPI	0.00403	-0.02435	0.04623	0.008867
Taiwan M2	0.02279	-0.00965	0.07965	0.017964
S.P.500	0.02099	-0.26431	0.18952	0.079425
NASDAQ	0.02523	-0.39653	0.39327	0.12065
Dow Jones	0.02284	-0.29199	0.19523	0.07738

We can see that the Taiwan stock exchange is more volatile than its US counterpart based on its standard deviation and the difference between the minimum and maximum quarterly return, but all indexes yield positive average returns over the examined period. The GDP index shows robust growth for Taiwan with no significant inflation risks (CPI). In the Appendix, the plots yield a better picture of the performance of the Taiwan economy from the fourth quarter of 1984 to the second quarter of 2015.

4 Linear Model Specification and Estimation

We start with a linear model, where each variable is included together with its own eight lags, as in Aslanidis *et al.* (2002), and where the natural logarithm of each variable has been differenced to achieve stationarity. Since we have 72 variables to consider in total, we use some automatic procedures to select the number of regressors that fit the data best, statistically. The first method we use is a stepwise regression. Stepwise regression is an iterative procedure where one starts with no regressors and then sequentially adds the regressors with the highest contribution, as measured by the Akaike information criterion (AIC). After adding each new variable, the procedure removes any variable that no longer provides an improvement in the model fit, and finally, the best subset of regressors is selected. The procedure described is called a stepwise regression with both forward and backward selection; see Gareth *et al.* (2014) for details.

We apply this methodology to our data each time using a different index for the US stock market. Table 2 displays the results.

Since the specification with the lower AIC is the preferred one, the NASDAQ and the Dow Jones Index are selected, but notice that in this case, the stepwise regression selects too many regressors, 34 and 27 respectively in total. Fitting a non-linear model with so many regressors can lead to overfitting the data, hence lead to poor forecasting power.

Table 2. Stepwise Regression

US Index	AIC	Number of regressors selected
Dow Jones	-527.04	34
NASDAQ	-532.73	27
S.P.500	-516.24	33

Therefore, we switch to another statistical procedure that usually selects for a smaller set of regressors. The LASSO (Least Absolute Shrinkage and Selection Operator) seeks to select the most informative regressors by adding a penalty term while minimizing the residual sum of squares. This penalty term forces some of the coefficients to be exactly zero; therefore, LASSO can perform variable selection and parameter estimation simultaneously. For details, see Tibshirani (1996) and Gareth *et al.* (2014).

Mathematically, we seek to estimate the coefficients' vector $\hat{\beta}_L$ by minimizing the following:

$$\underbrace{\sum_{t=1}^{n}(y_t - \beta_0 - \sum_{j=1}^{p}\beta_j x_{tj})^2}_{RSS} + \underbrace{\lambda\sum_{j=1}^{p}|\beta_j|}_{Penalty\ term}$$

where p is the number of parameters to be estimated, and λ is a tuning parameter, the magnitude of which determines how many regressors will be left out. It needs to be estimated, and in our approach, we use cross- validation.

On table 3 we produce the results of applying the above methodology to pick the "best" linear model, each time using a different US Stock Market index. In particular, we display the number of regressors selected and the adjusted R-squared statistic, the AIC, BIC, and Root Mean Square Error (RMSE) of each linear model fitted with the regressors picked by the LASSO methodology.

Table 3. LASSO Selection

US Index used	Number of regressors selected	Adj. R^2	AIC	BIC	RMSE
Dow Jones	11	0.6003	-138.3493	-101.2729	0.127336
NASDAQ	10	0.5919	-136.5765	-102.3521	0.129226
S.P.500	12	0.5682	-129.3602	-95.13587	0.132921

Based on the above results, we continue the analysis by using the Dow Jones Index representing the US stock market and the regressors picked using LASSO: the second and fifth lag of the TAIEX return (TAIEX2, TAIEX5), Tai- wan GDP and its third lag (TWGDP, TWGDP3), CPI and its fourth lag (TWCPI, TWCPI4), the second differenced M2 variable (TWM22), the fourth differenced Taiwan/US dollar exchange rate (TWUSD4), the second difference of the risk-free rate (TWRATE2), and the Dow Jones Index together with its fourth lag (DOWJONES, DOWJONES4). The linear regression results are shown in table 4;

Table 4. Linear Model Estimates

Variable	Estimate	Standard error	p-value
Intercept	-0.01935	0.02356	0.4130
TAIEX2	0.35914	0.05983	2.27e-08***
TAIEX5	0.13488	0.04420	0.0028***
TWGDP	1.6849	0.8245	0.04325**
TWGDP3	-0.87914	0.5826	0.1340
TWCPI	-5.8953	1.6674	0.00058***
TWCPI4	2.2642	1.0477	0.0327**
TWM22	0.1230	0.5537	0.8245
TWUSD4	0.2386	0.2204	0.2811
TWRATE2	-0.1206	0.0623	0.055*
DOWJONES	0.5687	0.1989	0.0050***
DOWJONES4	-0.0737	0.1227	0.5492
Adjusted R-squared	0.6003		
F-statistic	18.34 (11 and 116 DF)		<2.2e-16
LM test	36.4288		7.102044e-05
Box-Pierce test	55.27		3.741e-05

The results suggest that the endogenous effect of the stock market is positive and statistically significant after two and five quarters, while the US stock market has a positive contemporaneous effect that becomes negative but not statistically significant after four quarters. There is a strong contemporaneous effect of economic activity on the stock market as measured by GDP that becomes negative after three quarters, and the same pattern is observed for the CPI. TWM22 and TWUSD4 show no statistically significant effect, but we have mentioned that there is strong statistical evidence to retain all the regressors in our model based on the statistical work we did before as well as the F-statistic value of this regression.

The model fit is decent with 60% variation in the stock market return explained, but there is evidence of remaining serial correlation and ARCH effects. It is important to correct for these effects in the non-linear model developed next.

5 Non-linear Model Specification and Estimation

To select the best fitting non-linear model, we follow the specific-to-general procedure as recommended by Granger (1993). First, after specifying the appropriate linear model and its order p (if it is an AR(p)) for our time-series data, we test the null hypothesis of linearity against the alternative of SETAR-type nonlinearity, and we select the appropriate threshold variable that determines the regimes. Then, we estimate the parameters of the selected model and evaluate it using diagnostic tests. If necessary, we modify the model and finally, use it for descriptive and forecasting purposes.

The linear model has been selected in the previous section to test whether there is non-linearity in our overall relationship between the Taiwan stock market with its lags and all the other regressors. We use the linearity test as developed by Luukkonen *et al.* (1988), using a third-order Taylor approximation of the smooth transition function. The test comes in two variants; an LM (Lagrange Multiplier) test that is chi-squared distributed and an F-version test that has better power properties at small sample sizes. According to Terasvirta (1994), the nonlinearity tests can be used to select the threshold variable among the candidates. For details, see Chapter 3 of Franses and van Dijk (2000). The auxiliary regression needed for the LM nonlinearity test is now given as,

$$y_t = \beta_0^T y_{t,p} + \beta_1^T y_{t,p} * s_t + \beta_2^T y_{t,p} * S_t^2 + \beta_3^T y_{t,p} * S_t^3 + \epsilon_t \tag{5}$$

where s_t is the candidate for the threshold variable, β_j, j=0,1,2,3, the parameter vector, and in our case, $y_{t,p}$ contains all the regressors used in our linear specification, not only lags of TAIEX. Hence, it is a smooth-transition regression model, not a smooth-transition autoregressive model. In table 5, we display the LM and F-statistic together with their p-values, trying different lags of the TAIEX and the Dow Jones index, respectively, as candidates for the threshold variable s_t.

Table 5. LM and F-statistics for Nonlinearity

Variable	LM-statistic	p-value	F-statistic	p-value	Decision (linear model)
TAIEX2	15.169	0.00167	5.5569	0.00130	Reject
TAIEX3	7.4779	0.05812	2.5645	0.0577	Fail to Reject
Dow Jones	16.5407	0.00087	6.1339	0.00063	Reject
Dow Jones 2	12.2145	0.00668	4.3603	0.00589	Reject
Dow Jones 3	4.6380	0.2003	1.5540	0.20398	Fail to Reject

The above table finds strong evidence of nonlinearity and also suggests using the Dow Jones Index as the threshold (switching) variable. Hence, this is the next step to fit a smooth transition regression model to our time series. The specification is selected using a logistic transition function and models the volatility as a GARCH(1,1) process. Results for the estimated parameters and other diagnostic statistics are displayed in Table 6.

From Table 6 we can see that the change-point is statistically significant and near zero ($\hat{q}$=0.0189), approximately halfway between positive and negative returns in agreement with the existence of two separate regimes in the Taiwan stock market. This becomes clearer looking at Figure 3, where we plot the transition function together with the probability of changing from one regime to another. The value of $\hat{\gamma}$=32.18 shows that the switching between the two regimes happens fast but not as fast as using an indicator variable to model it, so the smooth transition model is justified in analyzing our dataset. The effect of the lagged TAIEX values is large and statistically significant at the low-return regime while the contemporaneous effect is significant at the high return and similar to the estimates suggested by the linear model. The effect of lagged economic activity is large and negative at the low-return (negative) regime but becomes positive, large in value, and significant at the high-return regime; a result, in a sense, being the average of the estimates from the linear model. The CPI effect is strong and positive with an equally strong negative effect only at the high-return regime, which makes sense since inflation shocks are not welcome in the stock market while the exchange rate exerts a positive effect in both regimes. The lagged M2 variable has a strong and positive effect only in the first regime, where the risk-free rate also has a negative effect on the stock market index in Taiwan. Finally, the Dow Jones Index exerts a positive contemporaneous influence in the Taiwan stock market return, again in the high-return regime, only turning negative after 4 quarters. The model has a good enough adjusted R-squared value, around 75%, and while there is some evidence of a remaining ARCH effect, by plotting the autocorrelation function, it does not seem serious enough to require further action[6].

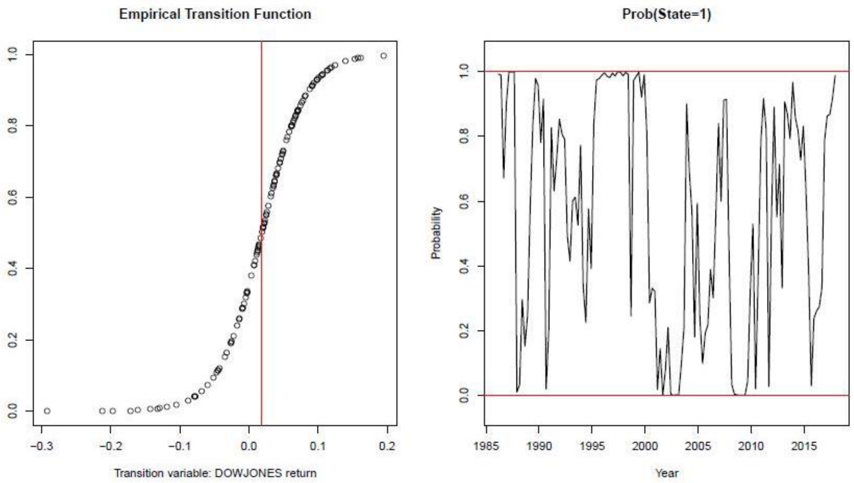

Figure 3. Transition Variable and States Probability

[6] Plots are available by requesting the author.

Table 6. Smooth Transition Model (Logistic) Estimates

	Regime 1			*Regime 2*		
Variable	Estimate	Std.Err.	p-value	Estimate	Std.Err.	p-value
Intercept	0.00438	0.0310	0.8876	-0.0694	0.03448	0.0438**
TAIEX2	0.5803	0.1029	0.0000***	0.3740	0.1080	0.0005***
TAIEX5	0.1925	0.0577	0.0008***	0.2256	0.1212	0.0626*
TAIEX7	-0.2030	0.0641	0.0015***	-0.6417	0.0835	0.0000***
TAIEX8	0.1244	0.0659	0.0593*	0.4391	0.1007	0.0000***
TWGDP	-2.4926	0.8653	0.0039***	3.4661	1.0150	0.0006***
TWCPI	-3.8518	2.2667	0.0892*	2.6886	3.0913	0.3844
TWM22	0.4532	0.4321	0.2942	0.1532	0.7266	0.8329
Dow Jones	0.3255	0.2408	0.1764	0.1408	0.2602	0.5882
Dow Jones 4	-0.0448	0.1522	0.7681	-0.2800	0.1294	0.0305
γ	61.717	25.843	0.0169**			
q	0.0139	0.0050	0.0061***			
LogLikelihood	118.7271					
$R2$ (adj)	0.8085					
AIC	-1.6474					
BIC	-1.0746					
Box-Pierce test (serial correlation)	57.744		1.582e-05			
ARCH LM-test	20.796		0.0534*			
LM-statistic (nonlinearity)	41.890		0.0731*			
F-statistic (nonlinearity)	1.4324		0.1071			

6 Forecasting Exercise

To see whether our nonlinear specification is capable of outperforming the simple linear specification in forecasting the TAIEX, we conduct a simple out-of-sample forecasting exercise predicting the TAIEX return for horizons four and eight quarters ahead. This is done by using the R package twinkle developed by Ghalanos (2014). As explained there, we choose the Monte Carlo method to conduct the 4- and 8-step ahead forecast. The 4-step ahead forecast is from the 3rd quarter of 2016 to the 3rd quarter of 2017 (1 year ahead) while the 8-step ahead forecast is from the 1st quarter of 2015 to the end of 2017 (2 years ahead). The confidence plots of both forecasts are shown below.

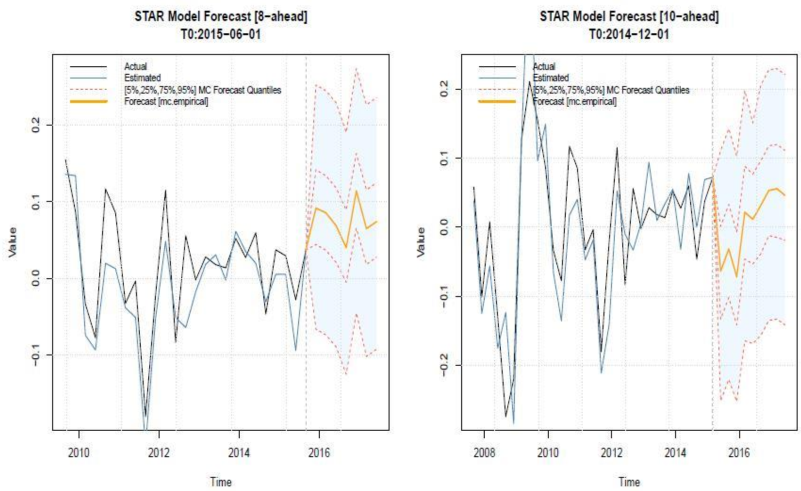

Figure 4. 4- and 8-step ahead Forecast

The measures for forecast accuracy evaluation employed are the following: Root Mean Squared Error (RMSE), Mean Forecast Error (MFE), Mean Absolute Error (MAE), Tracking Signal (TS), and Mean Absolute Percentage Error (MAPE). Although most are standard, a tracking signal[7] taking values between -4 and 4 implies that the model is producing accurate forecasts. The results are in table 7.

[7] It is computed as follows: $TS = \frac{1}{MAE}\sum_{t=1}^{n}(x_t - f_t)$, where xt is the actual data point, and ft its forecast at time t.

Table 7. Forecast Accuracy Measures

Measure	Linear 4-ahead	Nonlinear 4-ahead	Linear 8-ahead	Nonlinear 8-ahead
RMSE	0.01962	0.02687	0.01328	0.01965
MFE	0.006618	-0.02788	0.00771	-0.00349
MAE	0.03424	0.03586	0.03202	0.04783
TS	0.7729	-3.1100	1.9251	-0.5845
MAPE	1.8273	2.9551	1.6236	2.0178

The effects are mixed. The nonlinear model does not seem to outperform the linear one in most measures (especially in the one-year-ahead prediction) but gets better at the longer forecasting horizon. It seems that based on the Mean Forecast Error, the linear model tends to under-forecast while the nonlinear smooth transition model tends to over-forecast. Overall, the results are similar for the 8-step ahead forecasting exercise for both the linear and nonlinear models, but keep in mind that the linear model was strongly rejected. Therefore, in our specific time period and dataset, we are not able to confirm that nonlinear models to perform better (even marginally) in forecasting than linear ones, as has been found in other studies; see McMillan (2001), Bredin *et al.* (2005) and Guidolin *et al.* (2014).

7 The Great Financial Crisis Effect

We briefly state the linear and nonlinear model estimates by splitting our sample into two periods: before and after the Great Financial Crisis (GFC). We use the 3rd quarter of 2018 as the date this event began, using the National Bureau of Economic Research (NBER) on Business Cycles dating[8].

We start with the linear model results for the pre- and post-GFC period side by side.

[8] https://www.nber.org/cycles.html

Table 8. Linear Model Estimates: pre- and post-GFC.

	pre-GFC		*post-GFC*	
Variable	Estimate	p-value	Estimate	p-value
Intercept	-0.0236	0.5452	-0.0469	0.0706*
TAIEX2	0.3688	3.4e-06 ***	0.2492	0.0187**
TAIEX5	0.1476	0.0106**	0.0266	0.7781
TWGDP	1.7724	0.1042	1.2203	0.0998*
TWGDP3	-0.9747	0.2475	-0.3883	0.6042
TWCPI	-5.8086	0.0085***	-0.3020	0.8726
TWCPI4	2.6730	0.0647*	0.2319	0.8251
TWM22	0.1454	0.8501	1.0932	0.0860*
TWUSD4	0.2922	0.3257	0.1224	0.6217
TWRATE2	-0.2183	0.0816*	-0.1061	0.0019***
DOWJONES	0.6002	0.0391**	0.4375	0.0139**
DOWJONES4	-0.1373	0.4115	0.1159	0.2466
Adjusted R-squared	0.5786		0.8363	

Overall, there are differences between the two separate regimes in the sense that the effect of almost all variables on the Taiwanese stock market are larger in magnitude while the sign in front of each variable is different only for the DOWJONES4, which is not statistically significant anyway. Notice that the fit is much better in the post-GFC estimates, judging by the adjusted R-squared value of 84% versus only 58% in the pre-GFC model.

The nonlinear model results are displayed in the two tables below.

Table 9. Nonlinear Model Estimates: pre-GFC.

	Regime 1		Regime 2	
Variable	Estimate	p-value	Estimate	p-value
Intercept	-0.0470	0.4737	0.1682	0.0075***
TAIEX2	0.7262	0.0000***	0.1682	0.4492
TAIEX5	0.3267	0.0000***	-0.1484	0.5150
TWGDP	6.2000	0.0000***	-5.7616	0.3091
TWGDP3	-3.9492	0.0000***	0.7918	0.7203
TWCPI	-7.5895	0.0041***	-6.6646	0.1081
TWCPI4	7.1607	0.0010***	-2.5307	0.4421
TWM22	0.5279	0.6070	-0.9243	0.3163
TWUSD4	0.2016	0.5890	0.1512	0.6872
TWRATE2	-0.6069	0.0009***	0.3933	0.1858
DOWJONES	-0.3395	0.6080*	1.7475	0.0000***
DOWJONES4	-0.0533	0.8098	0.0010	0.9961
γ	22.988	0.01201**		
q	-0.0233	0.61508		
LogLikelihood	79.1132			
R2 (adj)	0.7752			
AIC	-1.1886			
BIC	-0.3896			

Table 10. Nonlinear Model Estimates: post-GFC.

	Regime 1		Regime 2	
Variable	Estimate	p-value	Estimate	p-value
Intercept	-0.2742	0.0552*	-0.1199	0.0000***
TAIEX2	0.1348	0.6803	0.2677	0.0419**
TAIEX5	1.8876	0.0266**	-0.3668	0.0000***
TWGDP	-23.5782	0.0225**	0.6441	0.1505
TWGDP3	6.3468	0.0323**	1.7413	0.0002***
TWCPI	25.1946	0.0237**	1.5938	0.2665
TWCPI4	-0.6105	0.8731	-0.5443	0.5106
TWM22	7.0149	0.1279	1.9137	0.0000***
TWUSD4	5.1571	0.0515*	-0.7975	0.0009***
TWRATE2	-0.2975	0.0001***	-0.1873	0.0000***
DOWJONES	0.2473	0.6364	0.5335	0.0050***
DOWJONES4	-0.3824	0.1751	0.2053	0.0000***
γ	174.0350	0.0002***		
q	0.10054	0.0000****		
LogLikelihood	105.5353			
R2 (adj)	0.8927			
AIC	-3.9762			
BIC	-2.7818			

What we observe is that in the pre-GFC data, all variables exert a significant effect on the TAIEX but only in the first regime (low-return) with the exception of the DOWJONES, which stays significant in both regimes. This is in contrast to the post-GFC data fit, where the effect of most variables is strong both in magnitude and statistically in the second regime. The pre-GFC data fit shows the change between the two regimes to occur slowly ($\hat{\gamma}$=22.988) with the change point in negative territory ($\hat{q}$=-2.33%). In the post-GFC data, the fit is better, judging by all the measures (Log-Likelihood, AIC, BIC, and R^2 adj.), and the regime change occurs very fast (($\hat{\gamma}$=174.03)). The change-point is located at a plus 10% return, signifying the remarkable TAIEX performance after the GFC. The plot of the transition functions below tells the story much better.

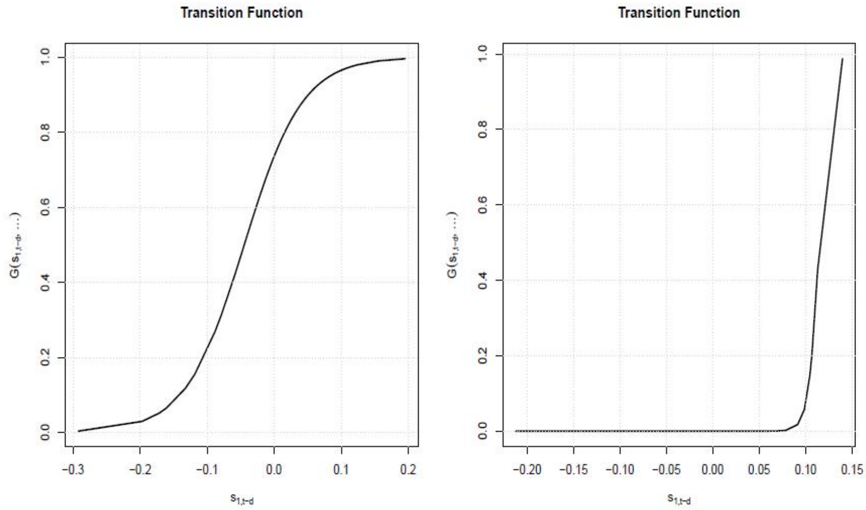

Figure 5. Pre- and Post-GFC Transition Functions

8 Conclusions

We employ a two-regime, smooth-transition regression model with a logistic transition function to measure the degree of the nonlinear interaction between the Taiwanese stock market and macroeconomic indexes for economic growth (GDP), price stability (CPI), money growth (M2), risk free rate (Taiwan T-Bills Rate), Taiwan dollar/US dollar exchange rate, and the Dow Jones, a US stock market index. As in Aslanidis *et al.* (2002), starting with eight lags for each variable, we employ the LASSO statistical methodology to pick 11 out of 72 possible regressors, and building on this specification, we find strong statistical evidence of nonlinear interactions with the Dow Jones index playing the role of the switching variable. The results of the fitted smooth transition model suggest two distinct bull and bear-type regimes for the stock market index with complex, significant, and asymmetric effects due to its lags and the GDP, CPI, M2, second differenced Taiwan risk-free rate, fourth differenced exchange rate, and lagged Dow Jones returns. In a simple 4- and 8-step ahead forecasting exercise, our nonlinear model does not seem to outperform the linear specification for most forecasting accuracy measures employed; therefore, we are not able to confirm recent results by Guidolin *et al.* (2014), suggesting that nonlinear models forecast better. Finally, we employ additional analyses by fitting linear and nonlinear specifications by splitting our dataset into two periods, the pre- and post-Great Financial Crisis dated at the third quarter of 2008 by the NBER. The nonlinear effects are different between the two periods with the post-GFC fit being better in all statistical measures employed. More work needs to be done in forecasting, probably by utilizing the most recent forecasting literature, as summarized in the textbook treatment by Elliott and Timmermann (2016).

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Appendix

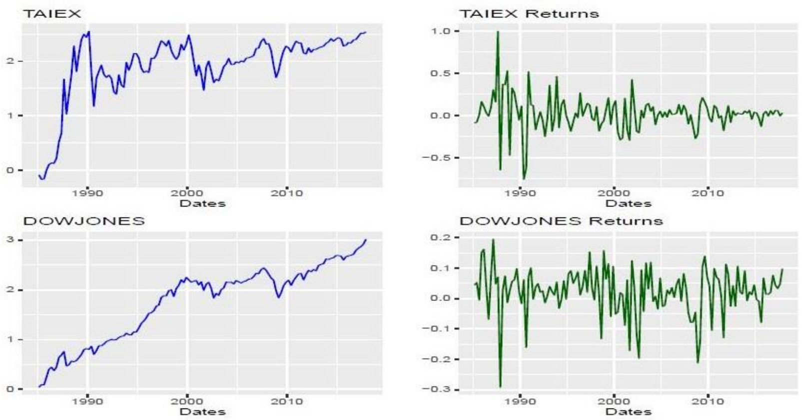

Figure 6. TAIEX and Dow Jones Indexes

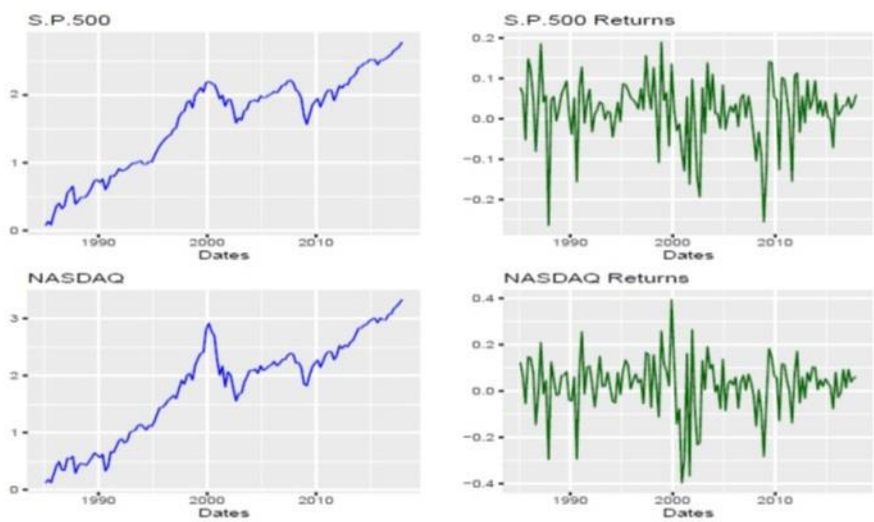

Figure 7. SP500 and NASDAQ Indexes

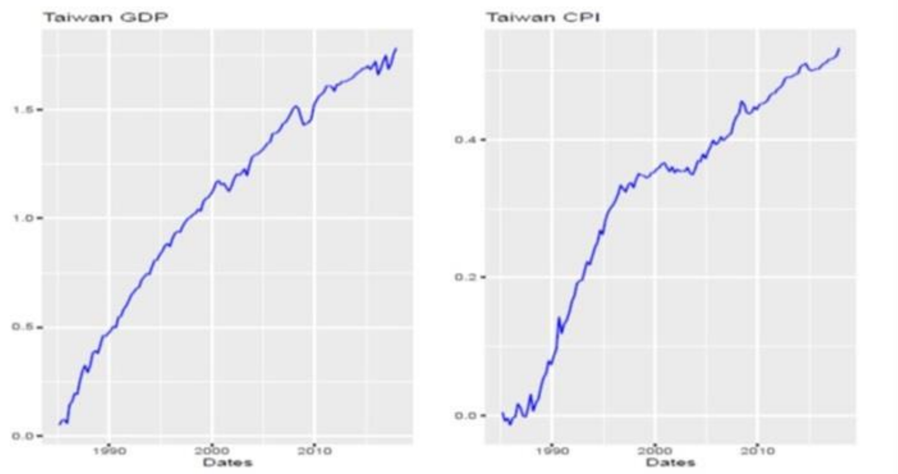

Figure 8. Taiwan GDP and CPI

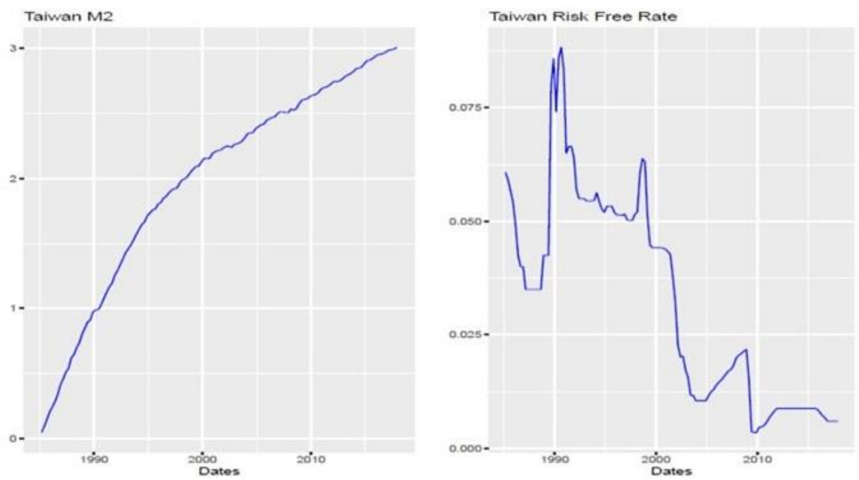

Figure 9. Taiwan M2 and Risk Free Rate

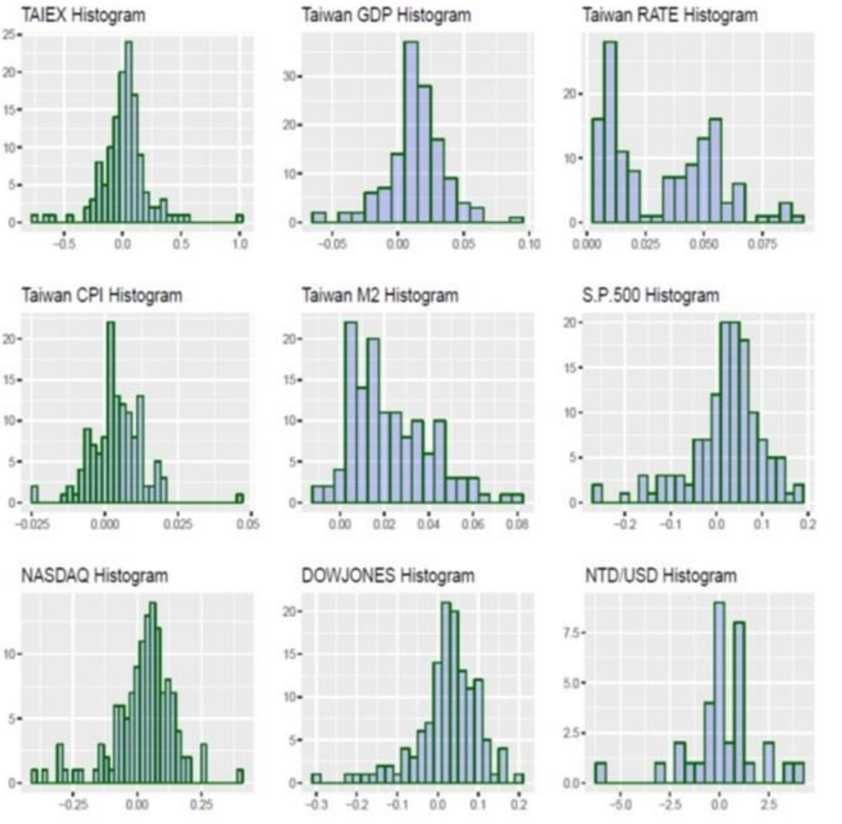

Figure 10. Data Histograms