Identifying the Dynamic Relationships among Four Pacific Rim Stock Markets by Bayesian Variable Selection

Feng-Chi Liu,* Cheng-Lin Tsai

Department of Statistics, Feng Chia University, Taiwan

Abstract

With the increasing frequency of international trade and cross-border financial flows, the relationships among financial markets have become increasingly close. This research thus employs a vector autoregressive (VAR) approach to assess the dynamic correlations of four Pacific Rim stock index returns and considers the identification problem of VAR models. We treat the identification problem of the VAR by considering restrictions on the VAR coefficients and adopt a Bayesian variable selection method to simultaneously estimate the model parameters and identify the possible subsets of variables. For the purpose of finding possible subsets of variables, we propose a coding method and a visualized approach. For illustration purposes, we consider the dynamic relationships among the returns of four stock market indices: S&P 500 of the U.S., Hang Seng Index of Hong Kong, Nikkei 225 of Japan, and Taiwan Capitalization Weighted Stock Index of Taiwan. We further employ three time periods of datasets to investigate the dynamic changes of the relationships among the four indices and find that there exists more complicated relationships among four indices during the global financial crisis of 2007-2009 and the period from 2016 to 2019 versus the relationships during the period from 2011 to 2015.

Keywords: Vector autoregressive model, Stochastic search variable selection, Bayesian inference, Markov chain Monte Carlo sampling.

JEL Classifications: C11, C58.

* Corresponding author.

E-mail address: fcliu@mail.fcu.edu.tw

1. Introduction

With the greater frequency of international trade and cross-border financial flows, market information is flowing more rapidly among various countries, bringing the relationships among financial markets of various countries closer. Traditionally, the univariate model treats countries as isolated from the rest of the global market, yet doing so may miss important information for capturing the interdependencies of different countries. Sims (1980) proposes a vector autoregressive (VAR) model to extend a univariate autoregressive model to a multivariate model. Subsequent studies in the literature prove that the VAR model is very useful for describing the dynamic behavior of economic and financial time series and for forecasting.

There are many applications of VAR models in modeling macroeconomics and financial time series. Hamilton (1994), Campbell, Lo and MacKinlay (1997), Cuthbertson (1996), Mills (1999), and Tsay (2001) all present applications of VAR models to financial data. In this paper we assess the dynamic relationships of four market index returns by a VAR approach and consider the identification problem of VAR models. While VAR models have characterized multivariate time series data and conducted macroeconomic forecasting in the past four decades, studies before the 2000s handle the issue of identification in VARs by actually ignoring it (Stock and Watson, 2001), but ignoring it results in an over-parameterized problem and typically causes adverse consequences on the precision of inference and the reliability of prediction. To overcome this problem, researchers consider the restrictions of VAR models. George, Sun and Ni (2008) propose a Bayesian variable selection mechanism to search for the possible restrictions of VAR models. They introduce a stochastic search variable selection (SSVS) prior proposed by George and McCulloch (1993) to doing shrinkage in the VAR models.

The SSVS method has been successfully used to many topics in time series modeling. For example, in univariate time series models, So, Chen and Liu (2006) consider the variable selection of autoregressive models with exogenous variables and generalized autoregressive conditional heteroscedasticity errors by the SSVS method. Further, Chen, Liu and Gerlach (2011) employ the SSVS method for the subset selection of threshold autoregressive moving-average models, and Yu *et al.* (2013) introduce the SSVS procedure for quantile regression based on the asymmetric Laplace distribution. In VAR models, Koop and Korobilis (2010) extend the method of SSVS to variants of time-varying parameter VAR (TVP-VAR) models. Koop (2013) constructs different sizes of VARs to examine their forecasting performance using U.S. macroeconomic data with 168 variables by the method of SSVS. Korobilis (2013) modifies the stochastic search algorithm of George *et al.* (2008) to adopt in nonlinear extensions of the VAR models. Feldkircher and Huber (2016) employ the SSVS prior put forward by George *et al.* (2008) to analyze international spillovers of U.S. shocks in a global VAR (GVAR) model. Crespo Cuaresma, Feldkircher and Huber (2016) consider a Bayesian GVAR (B-GVAR) model in macroeconomics forecasting and compare the predictive performance of B-GVAR models with a set of hierarchical priors.

Our study adopts the Bayesian SSVS method of George *et al.* (2008) in a VAR model to investigate the dynamic relationships among the returns of four Pacific Rim stock market indices: S&P 500 of the U.S., Hang Seng Index of Hong Kong, Taiwan Capitalization Weighted Stock Index of Taiwan, and Nikkei 225 of Japan. There are three separate time periods we consider for investigating the changes of their dynamics. The results show that the previous information of S&P 500 dominates the relationships of the other indices, and all four indices present different dynamic relationships with other indices in different time periods. We identify the dynamic relationship through a coding method and a visualized approach of Davis, Zang and Zheng (2016) in a real application. Thus, the main contributions of this paper are that we successfully employ a Bayesian variable selection method to obtain the restrictions on an over-parameterized VAR model, propose a coding method, and employ a visualized approach to identify the possible subsets of variables in the VAR model of the four Pacific Rim stock market indices.

The rest of this paper runs as follows. Section 2 provides a description of the VAR model and drives the Bayesian variable selection method. Section 3 presents the data and the considered time periods. Section 4 shows the results of variable selection in the VAR model. Finally, Section 5 concludes our findings.

2. Model and Methodology

2.1. Specification of Vector Autoregressive (VAR) Model

We present a vector autoregressive (VAR) model with *k* endogenous variables and *p* lagged effect in the following form:

$$\boldsymbol{y}_t = \mathbf{c} + \Phi_1 \boldsymbol{y}_{t-1} + \Phi_2 \boldsymbol{y}_{t-2} + \cdots + \Phi_p \boldsymbol{y}_{t-p} + \boldsymbol{\varepsilon}_t, \quad t = 1, 2, \dots, T, \tag{1}$$

where $\boldsymbol{y}_t = (y_{1,t}, y_{2,t}, \dots, y_{k,t})'$ is the variable of interest, and $\boldsymbol{\varepsilon}_t = (\varepsilon_{1,t}, \varepsilon_{2,t}, \dots, \varepsilon_{k,t})'$ is a vector of error term with zero-mean and variance-covariance matrix $\mathbf{\Sigma}_\varepsilon$. The constant term $\mathbf{c}$ and the coefficients Φ_i appear as follows:

$$\mathbf{c} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_k \end{bmatrix}, \quad \Phi_i = \begin{bmatrix} \phi_{11,i} & \phi_{12,i} & \cdots & \phi_{1k,i} \\ \phi_{21,i} & \phi_{22,i} & \cdots & \phi_{2k,i} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{k1,i} & \phi_{k2,i} & \cdots & \phi_{kk,i} \end{bmatrix}, \; i = 1, 2, \dots, p.$$

The literature has widely used the VAR model for capturing the dynamic relationship of multiple time series data and forecasting macroeconomic changes. Aggregating the VAR model (1) for all time $t = 1, 2, \dots, T$, we rewrite the VAR(*p*) model in matrix form:

$$\boldsymbol{Y} = \boldsymbol{\Phi X} + \boldsymbol{\epsilon}, \quad \boldsymbol{\epsilon} \sim N(0, \boldsymbol{\Sigma}_\epsilon), \tag{2}$$

where

$$\boldsymbol{Y} = [\boldsymbol{y}_1 \quad \boldsymbol{y}_2 \quad \cdots \quad \boldsymbol{y}_T] = \begin{bmatrix} y_{1,1} & y_{1,2} & \cdots & y_{1,T} \\ y_{2,1} & y_{2,2} & \cdots & y_{2,T} \\ \vdots & \vdots & \ddots & \vdots \\ y_{k,1} & y_{k,2} & \cdots & y_{k,T} \end{bmatrix}_{k\times T}, \qquad \boldsymbol{\Phi} = [\mathbf{c} \quad \Phi_1 \quad \cdots \quad \Phi_p]_{k\times(1+kp)},$$

$$\boldsymbol{X} = \begin{bmatrix} 1 & \cdots & 1 & \cdots & 1 \\ \boldsymbol{y}_0 & \cdots & \boldsymbol{y}_{t-1} & \cdots & \boldsymbol{y}_{T-1} \\ \boldsymbol{y}_{-1} & \cdots & \boldsymbol{y}_{t-2} & \cdots & \boldsymbol{y}_{T-2} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \boldsymbol{y}_{1-p} & \cdots & \boldsymbol{y}_{t-p} & \cdots & \boldsymbol{y}_{T-p} \end{bmatrix}_{(1+kp)\times T},$$

$$\boldsymbol{\epsilon} = [\boldsymbol{\varepsilon}_1 \quad \boldsymbol{\varepsilon}_2 \quad \cdots \quad \boldsymbol{\varepsilon}_T] = \begin{bmatrix} \varepsilon_{1,1} & \varepsilon_{1,2} & \cdots & \varepsilon_{1,T} \\ \varepsilon_{2,1} & \varepsilon_{2,2} & \cdots & \varepsilon_{2,T} \\ \vdots & \vdots & \ddots & \vdots \\ \varepsilon_{k,1} & \varepsilon_{k,2} & \cdots & \varepsilon_{k,T} \end{bmatrix}_{k\times T} \text{ and } \boldsymbol{\Sigma}_\epsilon = \begin{bmatrix} \sigma_{1,1} & \sigma_{1,2} & \cdots & \sigma_{1,k} \\ \sigma_{2,1} & \sigma_{2,2} & \cdots & \sigma_{2,k} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{k,1} & \sigma_{k,2} & \cdots & \sigma_{k,k} \end{bmatrix}.$$

For the example of k=3 and p=2, we formulate the VAR(2) model as follows:

$$\begin{bmatrix} y_{1,1} & y_{1,2} & \cdots & y_{1,T} \\ y_{2,1} & y_{2,2} & \cdots & y_{2,T} \\ y_{3,1} & y_{3,2} & \cdots & y_{3,T} \end{bmatrix}$$

$$= \begin{bmatrix} c_1 & \phi_{11,1} & \phi_{12,1} & \phi_{13,1} & \phi_{11,2} & \phi_{12,2} & \phi_{13,2} \\ c_2 & \phi_{21,1} & \phi_{22,1} & \phi_{23,1} & \phi_{21,2} & \phi_{22,2} & \phi_{23,2} \\ c_3 & \phi_{31,1} & \phi_{32,1} & \phi_{33,1} & \phi_{31,2} & \phi_{32,2} & \phi_{33,2} \end{bmatrix} \begin{bmatrix} 1 & \cdots & 1 & \cdots & 1 \\ y_{1,0} & \cdots & y_{1,t-1} & \cdots & y_{1,T-1} \\ y_{2,0} & \cdots & y_{2,t-1} & \cdots & y_{2,T-1} \\ y_{3,0} & \cdots & y_{3,t-1} & \cdots & y_{3,T-1} \\ y_{1,-1} & \cdots & y_{1,t-2} & \cdots & y_{1,T-2} \\ y_{2,-1} & \cdots & y_{2,t-2} & \cdots & y_{2,T-2} \\ y_{3,-1} & \cdots & y_{3,t-2} & \cdots & y_{3,T-2} \end{bmatrix}$$

$$+ \begin{bmatrix} \varepsilon_{1,1} & \varepsilon_{1,2} & \cdots & \varepsilon_{1,T} \\ \varepsilon_{2,1} & \varepsilon_{2,2} & \cdots & \varepsilon_{2,T} \\ \varepsilon_{3,1} & \varepsilon_{3,2} & \cdots & \varepsilon_{3,T} \end{bmatrix}, \qquad \boldsymbol{\epsilon} \sim N(0, \boldsymbol{\Sigma}_\epsilon), \quad \boldsymbol{\Sigma}_\epsilon = \begin{bmatrix} \sigma_{1,1} & \sigma_{1,2} & \sigma_{1,3} \\ \sigma_{2,1} & \sigma_{2,2} & \sigma_{2,3} \\ \sigma_{3,1} & \sigma_{3,2} & \sigma_{3,3} \end{bmatrix}.$$

In the above example, there are $k \times (1 + kp) + (k \times k + k)/2 = 27$ parameters that need to be estimated. This model contemporarily considers the interdependencies of multiple time series, but it also is a challenge for estimating such a huge number of VAR coefficients with large numbers of k and p. Hence, in addition to contemporarily modeling the interdependencies of variables by VAR models, we consider a Bayesian approach for model inference and estimate the parameters by Markov chain Monte Carlo (MCMC). This is the first objective of our study.

The second objective of our study is to identify important variables of the VAR model. While the problem of over-parameterization may cause adverse results for precise inference and reliable prediction, we aim to identify the restrictions for a large number of VAR coefficients to avoid the over-parameterized problem. Thus, we employ a stochastic search variable selection (SSVS) method to identify the important variables of VAR models, which can be treated as the issue of model selection. The study then obtains the estimations and restrictions of VAR coefficients simultaneously by introducing a hierarchical shrinkage prior on the coefficients. The next subsection provides details of the variable selection approach mentioned above.

2.2. Bayesian Variable Selection

Let $n = k \times (1 + kp)$, the total number of unknown VAR coefficients, and $\boldsymbol{\Theta} = (\theta_1, \theta_2, \dots, \theta_n)' = vec(\boldsymbol{\Phi})$. To take uncertainty of variable selection into account, we employ the stochastic search variable selection (SSVS) method proposed by George and McCulloch (1993) for regression models and by George *et al.* (2008) in the context of VAR models. They propose a hierarchical prior for $\boldsymbol{\Theta}$ given a latent variable $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \dots, \lambda_n)'$ as follows:

$$\boldsymbol{\Theta}|\boldsymbol{\lambda} \sim N_n(\mathbf{0}, \boldsymbol{DRD}),$$

where $\boldsymbol{\lambda}$ is a vector of 0-1 variables, $\boldsymbol{D} = \text{diag}(h_1, h_2, \dots, h_n)$ is a diagonal matrix with h_i's defined as:

$$h_i = \begin{cases} \tau_{0i} & \text{if } \lambda_i = 0 \\ \tau_{1i} & \text{if } \lambda_i = 1 \end{cases}, \quad i = 1, 2, \dots, n,$$

and τ_{0i} and τ_{1i} are pre-selected constants such that $\tau_{0i} < \tau_{1i}$. The matrix $\boldsymbol{R}$ is a preselected correlation matrix. Following the guidance of George and McCulloch (1993), we set $\boldsymbol{R} = \boldsymbol{I}$ as an identity matrix to reflect the apriori independent of VAR coefficients. Thus, the SSVS prior assumes a mixture normal prior on each element of $\boldsymbol{\Theta}$:

$$\theta_i|\lambda_i \sim (1 - \lambda_i)N(0, \tau_{0i}^2) + \lambda_i N(0, \tau_{1i}^2), \quad i = 1, 2, \dots, n.$$

There are many discussion and recommendations in the literature for selecting (τ_{0i}, τ_{1i}), which control the variances of mixture normal priors, such as George and McCulloch (1993), George and McCulloch (1997), and George *et al.* (2008). The basic idea is to set τ_{0i} small and τ_{1i} large such that θ_i is restricted to a small value when $\lambda_i = 0$ and unrestricted when $\lambda_i = 1$. As George *et al.* (2008) mention, we consider a semiautomatic strategy for the selection of (τ_{0i}, τ_{1i}). The combination of (τ_{0i}, τ_{1i}) is set as $(c_0\hat{\sigma}_{\theta_i}, c_1\hat{\sigma}_{\theta_i})$ to associate with the standard error of least squares estimate of θ_i. In our empirical application, we provide the results of variable selection for different combinations of c_0 and c_1 to investigate the influence of different SSVS prior settings. Figure 1 presents the mixture normal densities for the combination of $(\tau_{0i}, \tau_{1i}) = (1, 10)$.

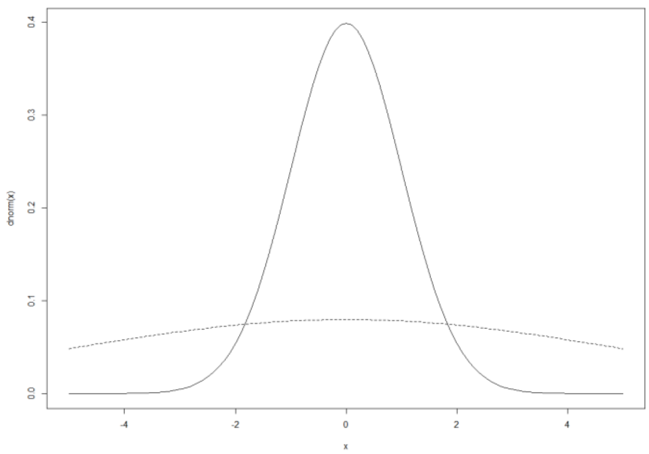


Figure 1: A Mixture of Normal Densities for $N(0, \tau_{0i}^2)$ (Solid Line) and $N(0, \tau_{1i}^2)$ (Dotted Line) with $(\tau_{0i}, \tau_{1i}) = (1, 10)$.

We now assume the elements of latent indicator $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_n)'$ to follow independent Bernoulli priors with inclusion probability p_i for the i^{th} coefficient.

$$\Pr(\lambda_i = 1) = p_i\,; \quad \Pr(\lambda_i = 0) = 1 - p_i, \qquad i = 1, 2, \ldots, n.$$

A natural choice of p_i is 0.5, implying that every variable is equally likely to enter the model. For the variance-covariance matrix of the error term, we set a Wishart prior for the inverse variance-covariance matrix $\boldsymbol{\Sigma}_{\epsilon}^{-1}$ as:

$$\boldsymbol{\Sigma}_{\epsilon}^{-1} \sim \mathbf{W}(\boldsymbol{S}^{-1}, \nu),$$

where $\boldsymbol{S}$ and ν are the hyperparameters of Wishart distribution. According to Koop and Korobilis (2010), we set $\boldsymbol{S} = \nu = 0$ to reflect a non-informative prior.

Under the specification of VAR model in (2), the likelihood function of the model is:

$$L(\boldsymbol{Y}|\boldsymbol{\Phi}, \boldsymbol{\Sigma}_{\epsilon}) \propto |\boldsymbol{\Sigma}_{\epsilon}|^{-T/2} \exp\left\{-\frac{1}{2}(\boldsymbol{Y} - \boldsymbol{\Phi}\boldsymbol{X})\boldsymbol{\Sigma}_{\epsilon}^{-1}(\boldsymbol{Y} - \boldsymbol{\Phi}\boldsymbol{X})'\right\}.$$

According to the Bayesian theorem, we conduct the conditional posteriors by the product of the likelihood function and the priors. The above setting of all priors involves conjugate priors. Thus, we obtain that their conditional posterior distributions have convenient forms as follows.

1. The conditional posterior distribution of $\boldsymbol{\Theta}$ given $\boldsymbol{\lambda}, \boldsymbol{\Sigma}_{\epsilon}, \boldsymbol{Y}$ is:

$$\boldsymbol{\Theta}|\boldsymbol{\lambda}, \boldsymbol{\Sigma}_{\epsilon}, \boldsymbol{Y} \sim N(\boldsymbol{\mu}_{\boldsymbol{\Theta}}, \boldsymbol{\Sigma}_{\boldsymbol{\Theta}}),$$

   where $\boldsymbol{\Sigma}_{\boldsymbol{\Theta}} = \{\boldsymbol{\Sigma}_{\epsilon}^{-1}(\boldsymbol{X}'\boldsymbol{X}) + (\boldsymbol{D}\boldsymbol{D})^{-1}\}^{-1}$, $\boldsymbol{\mu}_{\boldsymbol{\Theta}} = \boldsymbol{\Sigma}_{\boldsymbol{\Theta}}\{(\boldsymbol{\Psi}\boldsymbol{\Psi}')(\boldsymbol{X}'\boldsymbol{X})\hat{\alpha}\}$, and $\hat{\alpha} = vec((\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{Y})$.

2. The conditional posterior distribution of $\lambda_i, i = 1, 2, \ldots, n$, is:

$$\Pr(\lambda_i = 1|\mathbf{\Theta}, \boldsymbol{Y}) = \overline{p}_i\,; \qquad \Pr(\lambda_i = 0|\mathbf{\Theta}, \boldsymbol{Y}) = 1 - \overline{p}_i\,,$$

where

$$\overline{p}_i = \frac{\frac{1}{\tau_{1i}}\exp\left\{-\frac{\theta_i^2}{2\tau_{1i}^2}\right\}p_i}{\frac{1}{\tau_{1i}}\exp\left\{-\frac{\theta_i^2}{2\tau_{1i}^2}\right\}p_i + \frac{1}{\tau_{0i}}\exp\left\{-\frac{\theta_i^2}{2\tau_{0i}^2}\right\}(1-p_i)}.$$

3. The conditional posterior of $\mathbf{\Sigma}_\epsilon^{-1}$ is:

$$\mathbf{\Sigma}_\epsilon^{-1}|\,\mathbf{\Theta}, \boldsymbol{Y} \sim \mathbf{W}\left(\widehat{\boldsymbol{S}}^{-1}, \hat{\nu}\right),$$

where $\hat{\nu} = T + \nu$ and $\widehat{\boldsymbol{S}} = \boldsymbol{S} + (\boldsymbol{Y} - \mathbf{\Phi}\boldsymbol{X})(\boldsymbol{Y} - \mathbf{\Phi}\boldsymbol{X})'$.

We finally iteratively simulate the estimation of parameters from the conditional posteriors by the Markov chain Monte Carlo (MCMC) method. Since the conditional posteriors of all parameters have close forms, we can iteratively employ a simplified Gibbs sampler outlined in George *et al.* (2008) to draw the estimates of coefficients $\mathbf{\Theta}$ from a multivariate Normal distribution, the latent indicators $\boldsymbol{\lambda}$ from independent Bernoulli distributions, and the variance-covariance of error term $\mathbf{\Sigma}_{\boldsymbol{\epsilon}}$ from a Wishart distribution. The MCMC sampling algorithm repeats *N* times. We discard the first *M* iterates as burn-in iterations and compute the parameter estimates by averaging the last (*N*-*M*) draws to obtain the posterior estimates of parameters. For the purpose of variable selection, we calculate the posterior inclusion probability (PIP), which is the average of MCMC draws of λ_i, for each variable to identify important variables.

3. Data Description

For empirical application, we consider the daily data of returns on four Pacific Rim stock market indices: S&P 500 of the U.S., Nikkei 225 of Japan, Hang Seng Index (HSI) of Hong Kong, and Taiwan Capitalization Weighted Stock Index (TAIEX) of Taiwan. The datasets for the levels of the four indices are from Yahoo Finance via the R package "`quantmod`". We compute the return r_t at time *t* by:

$$r_t = [\ln(p_t) - \ln(p_{t-1})] \times 100,$$

where p_t is the price of stock index at time *t*.

The whole time span of the four index returns is from January 2007 to December 2019, and we separate them into three periods. For the choice of time periods in the empirical application, we would like to capture different behaviors among these three. The first period (Period I) is from January 5, 2007 to December 30, 2010 and includes the beginning of the U.S. subprime mortgage crisis in 2007, the subsequent 2008 global financial crisis, and the European sovereign debt crisis from the end of 2009 to all of 2010. This period is a highly volatile one for the world economy, and so we call it the global financial crisis (GFC) period. The second period (Period II) is chosen from January 5, 2011 to December 30, 2015 for capturing the behavior of the global economy recovering from the global financial crisis of 2007-2009. During this period, many countries use quantitative easing (QE) monetary policies to inject money into the economy to expand economic activity, such as the second and third rounds of quantitative easing (QE2 and QE3) of the U.S. in 2011 and 2012, the asset purchase program of the Bank of Japan in 2011, and the expanded asset purchase program of European Central Bank in 2015. These monetary policies are employed to recover from the severe recession of an individual country's economy. This is why we define the second period as the period of global economy recovering even though the global economy did not completely recover. The third period (Period III) is a recent one from January 5, 2016 to December 30, 2019. This period covers some important events that influence changes in the global economy such as the Bank of Japan moving into negative rate territory, the Brexit (the vote and follow-up withdrawal of the United Kingdom from the European Union), the inauguration of U.S. President Trump, and the U.S.-China trade war.

While there are significantly different economic situations during the three periods, we are interested in investigating the dynamic relationships of four stock indices. We adopt the Bayesian variable selection method introduced in Section 2 to figure out the dynamic relationships of the four market indices and to identify important variables in VAR models during the three periods. We perform the Bayesian SSVS method by the "`ssvs`" function of the R package "`bvartools`" and learn from Franz (2019) to implement the Bayesian MCMC sampling by using the "`bvartools`" package. The whole MCMC sampling sets up N=20,000 iterations, and burn-in iterations is M=10,000. We show the empirical results in the next section.

4. Empirical Results

4.1. Summary Statistics of Data

To investigate the dynamic relationships of the four market indices, our study considers an empirical VAR model with $k = 4$ endogenous variables and $p = 2$ lagged effects of all variables, formulated as in equation (3) below. In our method, we can set the lag p arbitrarily with a reasonable large integer, but we find that the PIP's are ignorable with low probabilities for high order of lags. Thus, we simply choose the lag $p = 2$ to reflect the features of quick delivery and short memory of financial market information. The notations SP_t, HS_t, TX_t, and NK_t respectively present the returns of S&P 500, HSI, TAIEX, and Nikkei 225 at time t.

$$\begin{cases} SP_t = c_1 + \phi_{11,1}SP_{t-1} + \phi_{12,1}HS_{t-1} + \phi_{13,1}TX_{t-1} + \phi_{14,1}NK_{t-1} + \\ \quad \phi_{11,2}SP_{t-2} + \phi_{12,2}HS_{t-2} + \phi_{13,2}TX_{t-2} + \phi_{14,2}NK_{t-2} + \varepsilon_{1,t} \\ HS_t = c_2 + \phi_{21,1}SP_{t-1} + \phi_{22,1}HS_{t-1} + \phi_{23,1}TX_{t-1} + \phi_{24,1}NK_{t-1} + \\ \quad \phi_{21,2}SP_{t-2} + \phi_{22,2}HS_{t-2} + \phi_{23,2}TX_{t-2} + \phi_{24,2}NK_{t-2} + \varepsilon_{2,t} \\ TX_t = c_3 + \phi_{31,1}SP_{t-1} + \phi_{32,1}HS_{t-1} + \phi_{33,1}TX_{t-1} + \phi_{34,1}NK_{t-1} + \\ \quad \phi_{31,2}SP_{t-2} + \phi_{32,2}HS_{t-2} + \phi_{33,2}TX_{t-2} + \phi_{34,2}NK_{t-2} + \varepsilon_{3,t} \\ NK_t = c_4 + \phi_{41,1}SP_{t-1} + \phi_{42,1}HS_{t-1} + \phi_{43,1}TX_{t-1} + \phi_{44,1}NK_{t-1} + \\ \quad \phi_{41,2}SP_{t-2} + \phi_{42,2}HS_{t-2} + \phi_{43,2}TX_{t-2} + \phi_{44,2}NK_{t-2} + \varepsilon_{4,t} \end{cases} \tag{3}$$

$$\begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \\ \varepsilon_{4,t} \end{bmatrix} \sim N(\mathbf{0}, \mathbf{\Sigma}_\varepsilon), \qquad \mathbf{\Sigma}_\varepsilon = \begin{bmatrix} \sigma_{1,1} & \sigma_{1,2} & \sigma_{1,3} & \sigma_{1,4} \\ \sigma_{2,1} & \sigma_{2,2} & \sigma_{2,3} & \sigma_{2,4} \\ \sigma_{3,1} & \sigma_{3,2} & \sigma_{3,3} & \sigma_{3,4} \\ \sigma_{4,1} & \sigma_{4,2} & \sigma_{4,3} & \sigma_{4,4} \end{bmatrix}$$

Table 1: Summary Statistics of the Four Market Index Returns for Three Periods.

	Mean	SD	Skewness	Excess Kurtosis	Min	Max	JB test p-value	LB test p-value
Period I: 2007/01/05~2010/12/30 (*T*=858)								
S&P 500	-0.040	1.696	-0.360	5.712	-9.470	10.246	< 0.01	0.002
HSI	0.015	2.154	0.078	6.230	-13.582	13.407	< 0.01	0.058
TAIEX	-0.007	1.572	-0.382	2.303	-6.735	6.525	< 0.01	0.351
Nikkei 225	-0.050	1.933	-0.224	7.086	-12.111	13.235	< 0.01	0.008
Period II: 2011/01/05~2015/12/30 (*T*=1,081)								
S&P 500	0.037	0.992	-0.584	5.170	-6.896	4.632	< 0.01	< 0.01
HSI	-0.007	1.202	-0.219	3.113	-6.018	5.519	< 0.01	0.587
TAIEX	-0.009	0.995	-0.317	2.933	-5.742	4.459	< 0.01	0.123
Nikkei 225	0.044	1.402	-0.689	6.079	-11.153	7.426	< 0.01	0.267
Period III: 2016/01/05~2019/12/30 (*T*=844)								
S&P 500	0.052	0.785	-0.898	3.930	-4.184	2.680	< 0.01	0.722
HSI	0.044	1.037	-0.348	1.699	-5.252	4.125	< 0.01	0.863
TAIEX	0.046	0.780	-1.151	8.569	-6.521	2.856	< 0.01	0.701
Nikkei 225	0.035	1.155	-0.494	6.610	-8.253	6.508	< 0.01	0.238

Table 1 shows the summary statistics of the four index returns for three periods. It is obvious that the four index returns have higher standard deviations during the volatile GFC period. Except for HSI in the first period, all indices present left-skewed distributions during the three periods. These phenomena reflect that the global economy has very slow growth in recent years. The Jarque–Bera (JB) tests for normality all reject the null hypotheses of normal assumption for the four indices in the three periods, meaning that all stock returns are not normally distributed. The excess kurtoses show the leptokurtic distribution for all series in three periods. The last column of Table 1 provides the p-values of Ljung-Box (LB) tests for series autocorrelations. The results show that S&P 500 exhibits significant autocorrelations both in

Period I and Period II. We also find the existence of autocorrelations in Nikkei 225 during Period I. Thus, it is worth it to consider a VAR model for capturing autocorrelations among the four indices.

4.2. Results of Variable Selection

To evaluate the results of variable selection, we monitor the selected subset of VAR variables in the MCMC sampling iterations, counting the selected subset of variables by a proposed coding method. For example, a vector of coefficients for three variables $(\phi_{11,1}, \phi_{21,1}, \phi_{31,1})'$ has a corresponding vector of latent indicators $(\lambda_1, \lambda_2, \lambda_3)$. If the latent indicators $(\lambda_1, \lambda_2, \lambda_3)$ are estimated as $(1, 1, 0)$, then it means that $\phi_{11,1}$ and $\phi_{21,1}$ are selected, but $\phi_{31,1}$ is not. Thus, we code the subset of variables for $(1, 1, 0)$ by $1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 6$. Furthermore, considering the model (3) in our study, we represent the model coefficients in matrix form as:

$$\boldsymbol{\phi} = \begin{bmatrix} \phi_{11,1} & \phi_{12,1} & \phi_{13,1} & \phi_{14,1} & \phi_{11,2} & \phi_{12,2} & \phi_{13,2} & \phi_{14,2} & c_1 \\ \phi_{21,1} & \phi_{22,1} & \phi_{23,1} & \phi_{24,1} & \phi_{21,2} & \phi_{22,2} & \phi_{23,2} & \phi_{24,2} & c_2 \\ \phi_{31,1} & \phi_{32,1} & \phi_{33,1} & \phi_{34,1} & \phi_{31,2} & \phi_{32,2} & \phi_{33,2} & \phi_{34,2} & c_3 \\ \phi_{41,1} & \phi_{42,1} & \phi_{43,1} & \phi_{44,1} & \phi_{41,2} & \phi_{42,2} & \phi_{43,2} & \phi_{44,2} & c_4 \end{bmatrix}.$$

There is a corresponding matrix of latent indicators $\boldsymbol{\Lambda}$ used to identify the selection of variables. Then, we define matrix $\boldsymbol{\Lambda}$ as:

$$\boldsymbol{\Lambda} = \begin{bmatrix} \lambda_1 & \lambda_5 & \lambda_9 & \lambda_{13} & \lambda_{17} & \lambda_{21} & \lambda_{25} & \lambda_{29} & \lambda_{33} \\ \lambda_2 & \lambda_6 & \lambda_{10} & \lambda_{14} & \lambda_{18} & \lambda_{22} & \lambda_{26} & \lambda_{30} & \lambda_{34} \\ \lambda_3 & \lambda_7 & \lambda_{11} & \lambda_{15} & \lambda_{19} & \lambda_{23} & \lambda_{27} & \lambda_{31} & \lambda_{35} \\ \lambda_4 & \lambda_8 & \lambda_{12} & \lambda_{16} & \lambda_{20} & \lambda_{24} & \lambda_{28} & \lambda_{32} & \lambda_{36} \end{bmatrix}.$$

We stack every three λ_i's by column sequence in $\boldsymbol{\phi}$ in one group and calculate a code number for each group by the above-mentioned coding method. Thus, for 36 coefficients in $\boldsymbol{\phi}$, we construct a code with 12 digits. For example, the number 750005001040 presented in Table 2 corresponds to matrix $\boldsymbol{\Lambda}$ as follows:

$$\boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

We select the coefficients $\phi_{11,1}$, $\phi_{21,1}$, $\phi_{31,1}$, $\phi_{41,1}$, $\phi_{22,1}$, $\phi_{44,1}$, $\phi_{21,2}$, $\phi_{33,2}$, and $\phi_{34,2}$, and thus the corresponding variables are the important variables in our VAR model.

Following the above coding technique, we record the selected subsets of variables and the sampled $\boldsymbol{\Lambda}$ in the after burn-in MCMC iteration. We count the frequency of the selected subsets of variables and calculate the posterior inclusion probability (PIP) for each variable. The PIP of the i^{th} variable is the proportion of λ_i that equals 1 during the collected MCMC iterations. We learn about the term PIP from Feldkircher and Huber (2016). They define PIP as a measure of a variable's importance in explaining the variation in the respective dependent variable. We compute the PIP's for all the variables by following steps.

Step 1: Collect the estimated $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_{36})$ for each iteration in the after burn-in iterations.

Step 2: Compute the percentages of λ_i's equal to one in the after burn-in iterations. These percentages are the PIP's defined by Feldkircher and Huber (2016).

The PIP's can be treated as the probabilities of variables that should be included in the model. Thus, a higher PIP means that the variable is more important for the model. For convenience, we plot the PIP's of all variables by a visualized approach as shown in Figures 2, 4 and 6. A large circle with dark blue color presents a high PIP of the corresponding variable. It also implies an important variable. Thus, we can easily identify important variables by the figures.

Tables 2, 4, and 6 list the results of subsets' selection for the three time periods by our coding method, with the top 5 selected subsets present for each combination of (c_0, c_1). We also provide the corresponding matrices $\boldsymbol{\Lambda}$ for helpfully understanding the possible relationship among the four indices. The probabilities presented in the parentheses of Tables 2, 4, and 6 are the posterior probabilities of the selected subsets during the collected MCMC iterations. They are all higher than the prior probability of each of the possible 2^{36} subsets, $1/2^{36} = 1/68{,}719{,}476{,}736$, indicating supportive evidence for the selected subsets. Focusing on the results of four combinations of c_0 and c_1, the combinations $(c_0, c_1) = (0.01, 20)$ and $(0.1, 20)$ seem likely to select more parsimonious models and provide higher selecting probabilities than other combinations, which can be obtained in Tables 2, 4, and 6. Thus, our inference mainly focuses on the results under $(c_0, c_1) = (0.01, 20)$ and $(0.1, 20)$.

In Tables 3, 5 and 7, we provide the results of parameter estimations under $(c_0, c_1) = (0.01, 20)$. The parameter estimations are based on the posterior draws of VAR coefficients Θ with variable selections. That is, the results are the average of estimated parameters, which are characterized by different degrees of shrinkage. Thus, it is common to obtain insignificant parameter estimates with variable selections when the corresponding variables do not present very strong influences, but still important, on the dependent variable. To illustrate the performance of parameter estimations, we also provide the results of parameter estimates without variable selection in Table 8 in the Appendix. The results show that our method can provide significant estimates for the coefficients of selected variables.

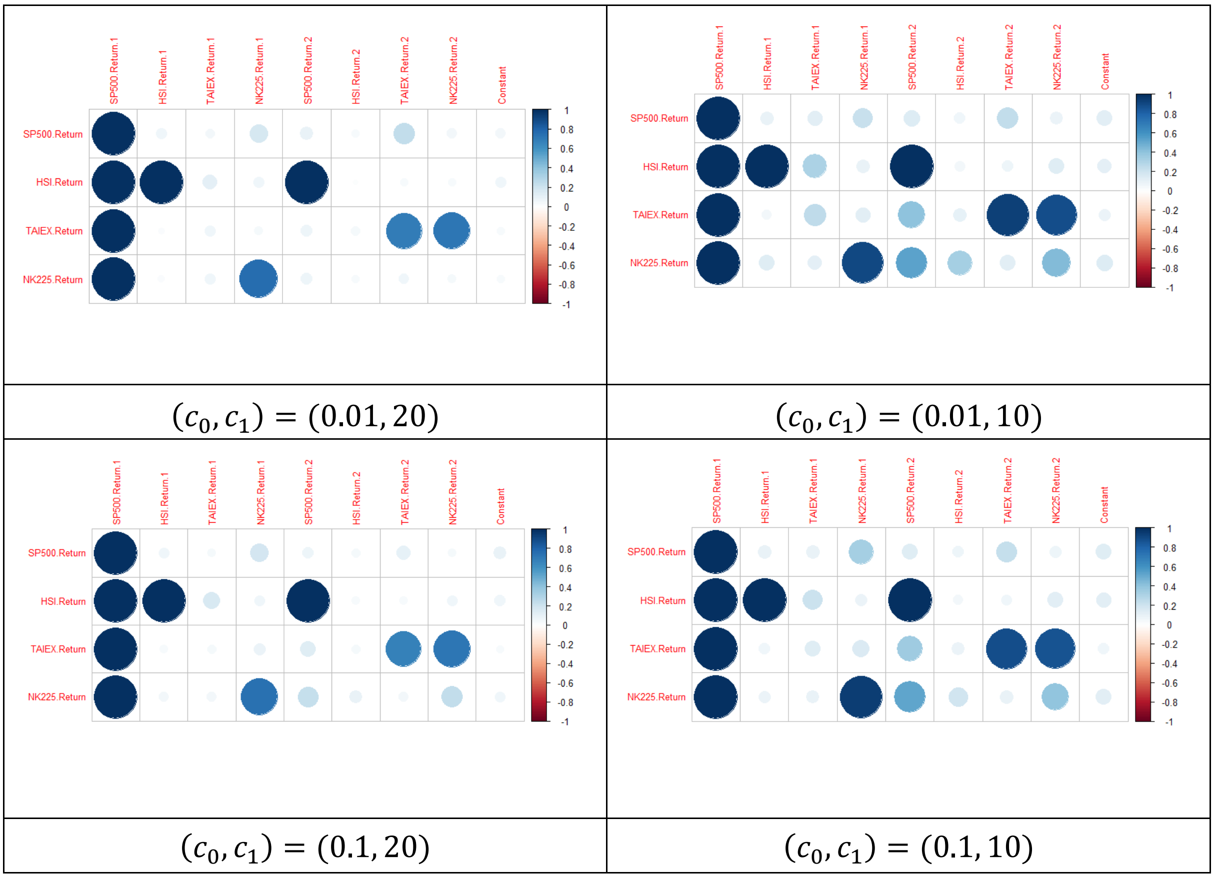


Figure 2: A Visualized Approach for Presenting PIP of Each Variable during Period I (from 2007 to 2010) under Four Combinations of (c_0, c_1).

We first concentrate on the results of Period I. Figure 2 shows that there are obviously some variables that should be included in the model, such as SP_{t-1} for the four indices, HS_{t-1} and SP_{t-2} for HSI, TX_{t-2} and NK_{t-2} for TAIEX, and NK_{t-1} for Nikkei 225. They apparently come with high PIP's. All constant terms seem not important in this model, and all indices are influenced by other indices except S&P 500. Furthermore, we obtain similar results from the subset selection results of Table 2. Observing the top 5 models for each (c_0, c_1), we obtain that our model should select the first two numbers "7" and "5", the sixth number "5", the ninth number "1", and the eleventh number "4" in our coding representation, which correspond to the coefficients of $\phi_{11,1}$, $\phi_{21,1}$, $\phi_{31,1}$, $\phi_{41,1}$, $\phi_{22,1}$, $\phi_{44,1}$, $\phi_{21,2}$, $\phi_{33,2}$ and $\phi_{34,2}$. We again demonstrate that the corresponding variables of these coefficients: SP_{t-1} for the four indices, HS_{t-1} and $SP_{t-2}$for HSI, TX_{t-2} and NK_{t-2} for TAIEX, and NK_{t-1} for Nikkei 225, are as the important variables in this dynamic relationships. Thus, we obtain that the previous information of S&P 500 dominates other indices during the GFC period and present a significant AR(1) relationship for S&P 500, HSI and Nikkei 225. Finally, we can probably summarize nine variables, SP_{t-1} for S&P 500, HSI, TAIEX, and Nikkei 225, HS_{t-1} for HSI, NK_{t-1} for Nikkei 225, SP_{t-2} for HSI, and TX_{t-2} and NK_{t-2} for TAIEX, as important variables in the VAR model for Period I.

Table 2: Results of Subsets' Selection for Period I (from 2007-2010).

(c_0, c_1)	$(\mathbf{0.1}, \mathbf{10})$	$(\mathbf{0.1}, \mathbf{20})$	$(\mathbf{0.01}, \mathbf{10})$	$(\mathbf{0.01}, \mathbf{20})$
Best	750005001040 (2.02%) $\begin{bmatrix}1&0&0&0&0&0&0&0&0\\1&1&0&0&1&0&0&0&0\\1&0&0&0&0&0&1&1&0\\1&0&0&1&0&0&0&0&0\end{bmatrix}$	750005001040 (5.62%) $\begin{bmatrix}1&0&0&0&0&0&0&0&0\\1&1&0&0&1&0&0&0&0\\1&0&0&0&0&0&1&1&0\\1&0&0&1&0&0&0&0&0\end{bmatrix}$	750005001040 (1.68%) $\begin{bmatrix}1&0&0&0&0&0&0&0&0\\1&1&0&0&1&0&0&0&0\\1&0&0&0&0&0&1&1&0\\1&0&0&1&0&0&0&0&0\end{bmatrix}$	750005000000 (6.66%) $\begin{bmatrix}1&0&0&0&0&0&0&0&0\\1&1&0&0&1&0&0&0&0\\1&0&0&0&0&0&0&0&0\\1&0&0&1&0&0&0&0&0\end{bmatrix}$
2nd Best	750005201040 (0.87%) $\begin{bmatrix}1&0&0&0&0&0&0&0&0\\1&1&0&0&1&0&0&0&0\\1&0&0&0&0&0&1&1&0\\1&0&0&1&1&0&0&0&0\end{bmatrix}$	750001001040 (2.97%) $\begin{bmatrix}1&0&0&0&0&0&0&0&0\\1&1&0&0&1&0&0&0&0\\1&0&0&0&0&0&1&1&0\\1&0&0&0&0&0&0&0&0\end{bmatrix}$	750005201040 (1.17%) $\begin{bmatrix}1&0&0&0&0&0&0&0&0\\1&1&0&0&1&0&0&0&0\\1&0&0&0&0&0&1&1&0\\1&0&0&1&1&0&0&0&0\end{bmatrix}$	750005001040 (5.55%) $\begin{bmatrix}1&0&0&0&0&0&0&0&0\\1&1&0&0&1&0&0&0&0\\1&0&0&0&0&0&1&1&0\\1&0&0&1&0&0&0&0&0\end{bmatrix}$
3rd Best	750005601040 (0.85%) $\begin{bmatrix}1&0&0&0&0&0&0&0&0\\1&1&0&0&1&0&0&0&0\\1&0&0&0&1&0&1&1&0\\1&0&0&1&1&0&0&0&0\end{bmatrix}$	750005201040 (1.76%) $\begin{bmatrix}1&0&0&0&0&0&0&0&0\\1&1&0&0&1&0&0&0&0\\1&0&0&0&0&0&1&1&0\\1&0&0&1&1&0&0&0&0\end{bmatrix}$	750005601040 (0.79%) $\begin{bmatrix}1&0&0&0&0&0&0&0&0\\1&1&0&0&1&0&0&0&0\\1&0&0&0&1&0&1&1&0\\1&0&0&1&1&0&0&0&0\end{bmatrix}$	750001001040 (5.20%) $\begin{bmatrix}1&0&0&0&0&0&0&0&0\\1&1&0&0&1&0&0&0&0\\1&0&0&0&0&0&1&1&0\\1&0&0&0&0&0&0&0&0\end{bmatrix}$
4th Best	750045001040 (0.80%) $\begin{bmatrix}1&0&0&1&0&0&0&0&0\\1&1&0&0&1&0&0&0&0\\1&0&0&0&0&0&1&1&0\\1&0&0&1&0&0&0&0&0\end{bmatrix}$	750045001040 (1.52%) $\begin{bmatrix}1&0&0&1&0&0&0&0&0\\1&1&0&0&1&0&0&0&0\\1&0&0&0&0&0&1&1&0\\1&0&0&1&0&0&0&0&0\end{bmatrix}$	770005001040 (0.78%) $\begin{bmatrix}1&1&0&0&0&0&0&0&0\\1&1&0&0&1&0&0&0&0\\1&0&0&0&0&0&1&1&0\\1&0&0&1&0&0&0&0&0\end{bmatrix}$	750045001040 (2.60%) $\begin{bmatrix}1&0&0&1&0&0&0&0&0\\1&1&0&0&1&0&0&0&0\\1&0&0&0&0&0&1&1&0\\1&0&0&1&0&0&0&0&0\end{bmatrix}$
5th Best	750005005040 (0.63%) $\begin{bmatrix}1&0&0&0&0&0&1&0&0\\1&1&0&0&1&0&0&0&0\\1&0&0&0&0&0&1&1&0\\1&0&0&1&0&0&0&0&0\end{bmatrix}$	750005000000 (1.63%) $\begin{bmatrix}1&0&0&0&0&0&0&0&0\\1&1&0&0&1&0&0&0&0\\1&0&0&0&0&0&0&0&0\\1&0&0&1&0&0&0&0&0\end{bmatrix}$	750045011060 (0.70%) $\begin{bmatrix}1&0&0&1&0&0&0&0&0\\1&1&0&0&1&0&0&0&0\\1&0&0&0&0&0&1&1&0\\1&0&0&1&0&1&0&1&0\end{bmatrix}$	750005005040 (2.38%) $\begin{bmatrix}1&0&0&0&0&0&1&0&0\\1&1&0&0&1&0&0&0&0\\1&0&0&0&0&0&1&1&0\\1&0&0&1&0&0&0&0&0\end{bmatrix}$

For the parameter estimations, since we find that the combination of $(c_0, c_1) = (0.01, 20)$ provides more reliable selection results with relative high selecting probabilities in Table 2, we only employ the results of parameter estimations under the combination of $(c_0, c_1) = (0.01, 20)$ in Table 3 for further inference. We see in Table 3 that SP_{t-1} has a significantly negative estimate on SP_t and positive estimates on HS_t, TX_t, and NK_t. Furthermore, HS_{t-1} and SP_{t-2} appear to have a negative estimate and a positive estimate respectively on HS_t, and TX_t is slightly but significantly influenced by NK_{t-2}. We consider the negative effect of previous stock returns as an alleviation effect (Li *et al.*, 2016). This phenomenon presents that the market will dampen extreme returns when market volatility tends to increase. These relationships also appear in Figure 3.

For Period II, the plots of PIPs in Figure 4 similarly show that SP_{t-1} for HSI, TAIEX, and Nikkei 225 and NK_{t-1} for Nikkei 225 are significantly important in this dynamic structure of VAR model. The other variables, NK_{t-1} for S&P 500, HSI and TAIEX, and HS_{t-2} for S&P 500, also show considerable PIP's in this period. Compared to Period I, the PIP of SP_{t-1} for S&P 500 falls during this period. Thus, comparing with Figure 2 and Figure 4, we obtain a more parsimonious model in Period II than in Period I by the plots of PIP's.

The results of subset selection in Table 4 present that the variable SP_{t-1} is still an important variable for HSI, TAIEX, and Nikkei 225, but SP_{t-1} seems not strongly necessary for S&P 500, while the first number "7" obtained in Period I is partially replaced by "3" in Period II as shown in Table 4. It means that the variable SP_t does not exhibit strong autocorrelation with the lagged variable SP_{t-1} during Period II. The variables HS_{t-1} for HSI and NK_{t-1} for Nikkei 225 again are obtained in the selected subsets, while the selected subsets under $(c_0, c_1) = (0.01, 20)$ and $(0.1, 20)$ present that the numbers "5" and "4" at the second and sixth digits are necessary. Other variables, NK_{t-1} for S&P 500 or HSI and HS_{t-2} for S&P 500, seem also to be necessary, while the numbers "4" and "2" at the fifth digit and the number "1" at the seventh digit are frequently obtained in the selected subsets in Table 4. Thus, we can probably summarize seven variables, SP_{t-1} for HSI, TAIEX, and Nikkei 225, HS_{t-1} for HSI, NK_{t-1} for Nikkei 225, NK_{t-1} for S&P 500, and HS_{t-2} for S&P 500, as important variables in the VAR model of Period II. The dynamic relationships of the four indices in Period II are more simple and parsimonious than the relationships in Period I.

Table 3: Results of Parameter Estimations of the Best Model under $(c_0, c_1) = (0.01, 20)$ for Period I (from 2007 to 2010).

Variable		S&P 500					HSI					TAIEX					Nikkei 225			
	Parameter	Mean	SD	95% C. I.		Parameter	Mean	SD	95% C. I.		Parameter	Mean	SD	95% C. I.		Parameter	Mean	SD	95% C. I.	
SP_{t-1}	$\phi_{11,1}$	**-0.133**	**0.034**	**-0.200**	**-0.065**	$\phi_{21,1}$	**0.518**	**0.041**	**0.437**	**0.599**	$\phi_{31,1}$	**0.341**	**0.030**	**0.281**	**0.399**	$\phi_{41,1}$	**0.558**	**0.035**	**0.490**	**0.626**
HS_{t-1}	$\phi_{12,1}$	-0.002	0.010	-0.038	0.001	$\phi_{22,1}$	**-0.183**	**0.029**	**-0.239**	**-0.122**	$\phi_{32,1}$	0.000	0.004	-0.001	0.001	$\phi_{42,1}$	0.000	0.007	-0.001	0.001
TX_{t-1}	$\phi_{13,1}$	0.001	0.012	-0.001	0.012	$\phi_{23,1}$	-0.008	0.028	-0.110	0.001	$\phi_{33,1}$	-0.002	0.011	-0.040	0.001	$\phi_{43,1}$	-0.002	0.014	-0.038	0.001
NK_{t-1}	$\phi_{14,1}$	-0.011	0.027	-0.094	0.001	$\phi_{24,1}$	0.002	0.012	-0.001	0.040	$\phi_{34,1}$	-0.001	0.007	-0.018	0.001	$\phi_{44,1}$	-0.063	0.041	-0.131	0.001
SP_{t-2}	$\phi_{11,2}$	-0.004	0.016	-0.062	0.001	$\phi_{21,2}$	**0.148**	**0.033**	**0.085**	**0.216**	$\phi_{31,2}$	0.003	0.013	-0.001	0.050	$\phi_{41,2}$	0.005	0.020	-0.001	0.079
HS_{t-2}	$\phi_{12,2}$	0.000	0.008	-0.001	0.001	$\phi_{22,2}$	0.000	0.003	-0.001	0.001	$\phi_{32,2}$	-0.001	0.007	-0.017	0.001	$\phi_{42,2}$	0.001	0.006	-0.001	0.001
TX_{t-2}	$\phi_{13,2}$	0.015	0.032	-0.001	0.107	$\phi_{23,2}$	0.000	0.006	-0.001	0.001	$\phi_{33,2}$	-0.073	0.056	-0.165	0.001	$\phi_{43,2}$	-0.002	0.011	-0.036	0.001
NK_{t-2}	$\phi_{14,2}$	0.000	0.009	-0.004	0.008	$\phi_{24,2}$	0.000	0.006	-0.001	0.001	$\phi_{34,2}$	**0.069**	**0.050**	**0.000**	**0.147**	$\phi_{44,2}$	-0.002	0.010	-0.035	0.001
	c_1	-0.003	0.019	-0.061	0.001	c_2	0.004	0.019	-0.001	0.077	c_3	0.000	0.007	-0.001	0.001	c_4	-0.002	0.012	-0.028	0.001
$\boldsymbol{\Sigma}_\varepsilon$	$\sigma_{1,1}$	2.843	0.138	2.587	3.130	$\sigma_{1,2}$	1.074	0.120	0.840	1.316	$\sigma_{1,3}$	0.520	0.087	0.352	0.693	$\sigma_{1,4}$	0.627	0.100	0.435	0.829
						$\sigma_{2,2}$	3.926	0.191	3.570	4.318	$\sigma_{2,3}$	1.798	0.117	1.580	2.038	$\sigma_{2,4}$	2.116	0.135	1.862	2.392
											$\sigma_{3,3}$	2.153	0.105	1.959	2.365	$\sigma_{3,4}$	1.403	0.097	1.222	1.599
																$\sigma_{4,4}$	2.861	0.139	2.605	3.149

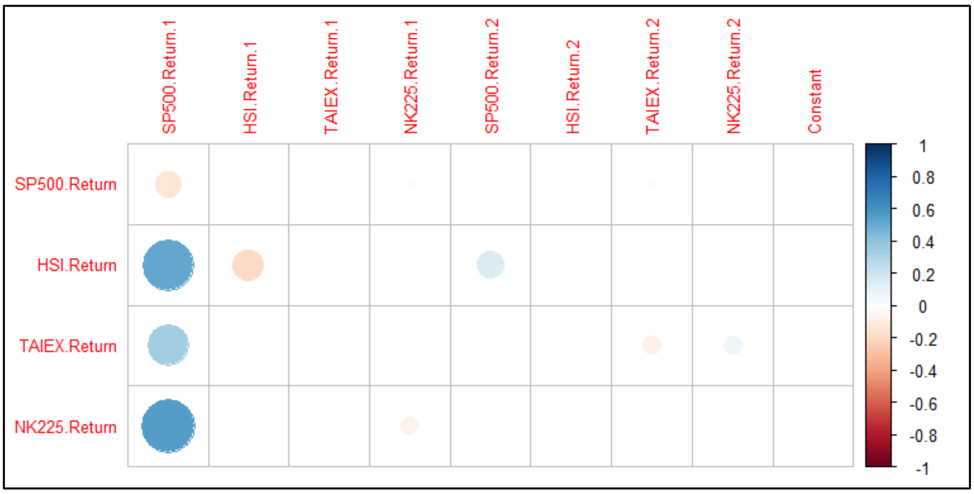


Figure 3: A Visualized Approach for Parameter Estimates of the Best Model under $(c_0, c_1) = (0.01, 20)$ during the Period I (from 2007 to 2010).

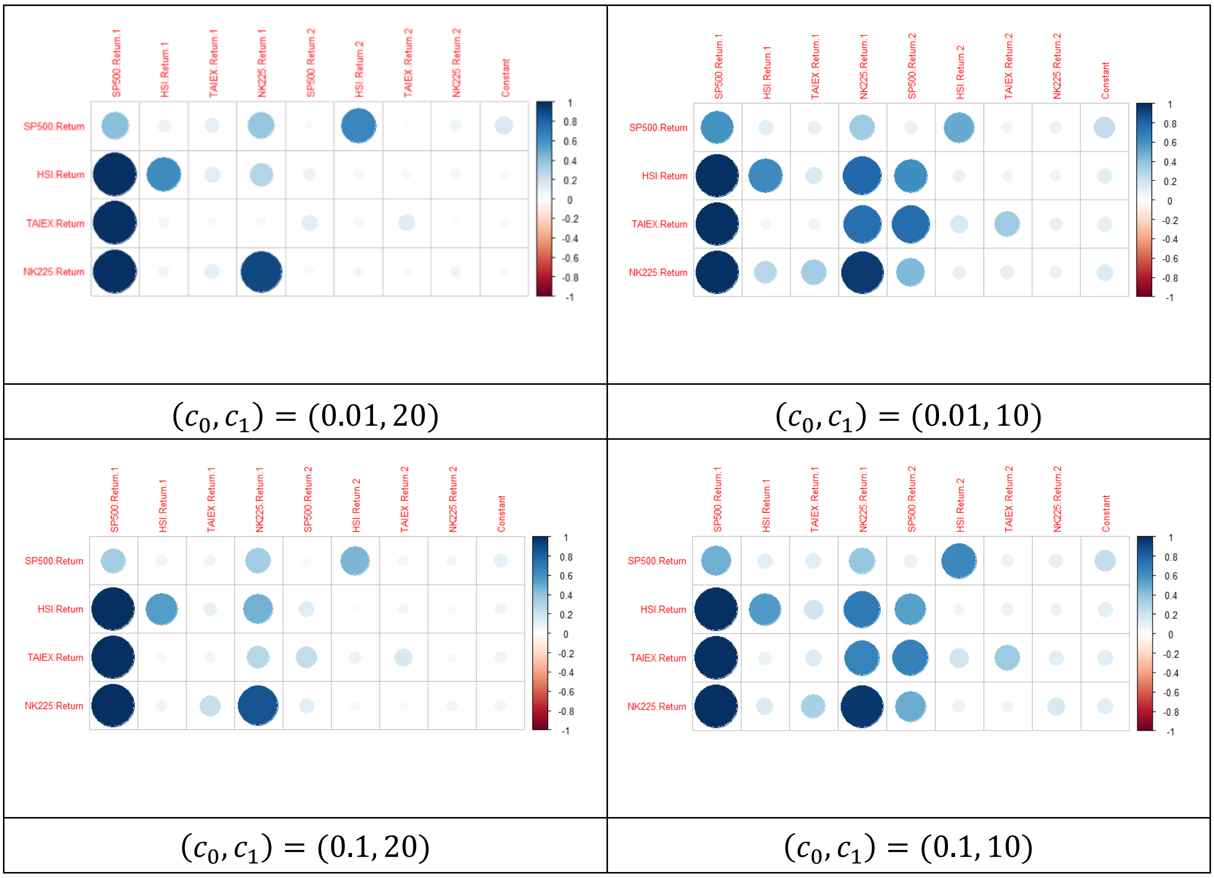


Figure 4: A Visualized Approach for Presenting PIP's of Each Variable during Period II (from 2011 to 2015) under Four Combinations of (c_0, c_1).

Table 4: Results of Subsets' Selection for Period II (from 2011 to 2015).

(c_0, c_1)	$(\mathbf{0.1}, \mathbf{10})$	$(\mathbf{0.1}, \mathbf{20})$	$(\mathbf{0.01}, \mathbf{10})$	$(\mathbf{0.01}, \mathbf{20})$
Best	750045600002(0.18%) $\begin{bmatrix}1&0&0&1&0&0&0&0&0\\1&1&0&0&1&0&0&0&0\\1&0&0&0&1&0&0&0&1\\1&0&0&1&1&0&0&0&0\end{bmatrix}$	750044000000(1.22%) $\begin{bmatrix}1&0&0&1&0&0&0&0&0\\1&1&0&0&0&0&0&0&0\\1&0&0&0&0&0&0&0&0\\1&0&0&1&0&0&0&0&0\end{bmatrix}$	742535400010(0.96%) $\begin{bmatrix}1&0&0&0&0&0&0&0&1\\1&0&1&1&1&0&0&0&0\\1&0&0&1&1&0&0&0&0\\1&1&1&1&0&0&0&0&0\end{bmatrix}$	350004000000(4.23%) $\begin{bmatrix}0&0&0&0&0&0&0&0&0\\1&1&0&0&0&0&0&0&0\\1&0&0&0&0&0&0&0&0\\1&0&0&1&0&0&0&0&0\end{bmatrix}$
2nd Best	750044100000(0.17%) $\begin{bmatrix}1&0&0&1&0&1&0&0&0\\1&1&0&0&0&0&0&0&0\\1&0&0&0&0&0&0&0&0\\1&0&0&1&0&0&0&0&0\end{bmatrix}$	350004000000(1.01%) $\begin{bmatrix}0&0&0&0&0&0&0&0&0\\1&1&0&0&0&0&0&0&0\\1&0&0&0&0&0&0&0&0\\1&0&0&1&0&0&0&0&0\end{bmatrix}$	750035500001(0.78%) $\begin{bmatrix}1&0&0&0&0&1&0&0&0\\1&1&0&1&1&0&0&0&0\\1&0&0&1&1&0&0&0&0\\1&0&0&1&0&0&0&0&1\end{bmatrix}$	750044100000(2.98%) $\begin{bmatrix}1&0&0&1&0&1&0&0&0\\1&1&0&0&0&0&0&0&0\\1&0&0&0&0&0&0&0&0\\1&0&0&1&0&0&0&0&0\end{bmatrix}$
3rd Best	350035600000(0.17%) $\begin{bmatrix}0&0&0&0&0&0&0&0&0\\1&1&0&1&1&0&0&0&0\\1&0&0&1&1&0&0&0&0\\1&0&0&1&1&0&0&0&0\end{bmatrix}$	340024000000(1.00%) $\begin{bmatrix}0&0&0&0&0&0&0&0&0\\1&0&0&1&0&0&0&0&0\\1&0&0&0&0&0&0&0&0\\1&0&0&1&0&0&0&0&0\end{bmatrix}$	770035400000(0.77%) $\begin{bmatrix}1&1&0&0&0&0&0&0&0\\1&1&0&1&1&0&0&0&0\\1&0&0&1&1&0&0&0&0\\1&0&0&1&0&0&0&0&0\end{bmatrix}$	350044101000(1.77%) $\begin{bmatrix}0&0&0&1&0&1&0&0&0\\1&1&0&0&0&0&0&0&0\\1&0&0&0&0&0&1&0&0\\1&0&0&1&0&0&0&0&0\end{bmatrix}$
4th Best	350004100000(0.16%) $\begin{bmatrix}0&0&0&0&0&1&0&0&0\\1&1&0&0&0&0&0&0&0\\1&0&0&0&0&0&0&0&0\\1&0&0&1&0&0&0&0&0\end{bmatrix}$	340024100000(0.94%) $\begin{bmatrix}0&0&0&0&0&1&0&0&0\\1&0&0&1&0&0&0&0&0\\1&0&0&0&0&0&0&0&0\\1&0&0&1&0&0&0&0&0\end{bmatrix}$	340034000010(0.61%) $\begin{bmatrix}0&0&0&0&0&0&0&0&1\\1&0&0&1&0&0&0&0&0\\1&0&0&1&0&0&0&0&0\\1&0&0&1&0&0&0&0&0\end{bmatrix}$	340004100000(1.58%) $\begin{bmatrix}0&0&0&0&0&1&0&0&0\\1&0&0&0&0&0&0&0&0\\1&0&0&0&0&0&0&0&0\\1&0&0&1&0&0&0&0&0\end{bmatrix}$
5th Best	340035700000(0.14%) $\begin{bmatrix}0&0&0&0&0&1&0&0&0\\1&0&0&1&1&0&0&0&0\\1&0&0&1&1&0&0&0&0\\1&0&0&1&1&0&0&0&0\end{bmatrix}$	350004100000(0.90%) $\begin{bmatrix}0&0&0&0&0&1&0&0&0\\1&1&0&0&0&0&0&0&0\\1&0&0&0&0&0&0&0&0\\1&0&0&1&0&0&0&0&0\end{bmatrix}$	350035600010(0.59%) $\begin{bmatrix}0&0&0&0&0&0&0&0&1\\1&1&0&1&1&0&0&0&0\\1&0&0&1&1&0&0&0&0\\1&0&0&1&1&0&0&0&0\end{bmatrix}$	340024100000(1.43%) $\begin{bmatrix}0&0&0&0&0&1&0&0&0\\1&0&0&1&0&0&0&0&0\\1&0&0&0&0&0&0&0&0\\1&0&0&1&0&0&0&0&0\end{bmatrix}$

Table 5: Results of Parameter Estimations of the Best Model under $(c_0, c_1) = (0.01, 20)$ for Period II (from 2011 to 2015).

Variable		S&P 500					HSI					TAIEX					Nikkei 225			
	Parameter	Mean	SD	95% C. I.		Parameter	Mean	SD	95% C. I.		Parameter	Mean	SD	95% C. I.		Parameter	Mean	SD	95% C. I.	
SP_{t-1}	$\phi_{11,1}$	-0.029	0.040	-0.117	0.001	$\phi_{21,1}$	**0.586**	**0.034**	**0.520**	**0.654**	$\phi_{31,1}$	**0.441**	**0.029**	**0.385**	**0.497**	$\phi_{41,1}$	**0.664**	**0.039**	**0.587**	**0.740**
HS_{t-1}	$\phi_{12,1}$	0.002	0.012	-0.001	0.045	$\phi_{22,1}$	-0.046	0.041	-0.121	0.001	$\phi_{32,1}$	-0.001	0.009	-0.018	0.001	$\phi_{42,1}$	0.002	0.012	-0.001	0.043
TX_{t-1}	$\phi_{13,1}$	0.004	0.017	-0.001	0.066	$\phi_{23,1}$	-0.009	0.026	-0.096	0.001	$\phi_{33,1}$	-0.001	0.007	-0.011	0.001	$\phi_{43,1}$	-0.012	0.038	-0.143	0.001
NK_{t-1}	$\phi_{14,1}$	**0.020**	**0.029**	**0.000**	**0.086**	$\phi_{24,1}$	-0.016	0.028	-0.088	0.001	$\phi_{34,1}$	-0.002	0.011	-0.040	0.000	$\phi_{44,1}$	-0.084	0.037	-0.144	0.000
SP_{t-2}	$\phi_{11,2}$	0.000	0.006	-0.001	0.001	$\phi_{21,2}$	0.005	0.022	-0.001	0.084	$\phi_{31,2}$	0.006	0.022	-0.001	0.080	$\phi_{41,2}$	0.003	0.017	-0.001	0.040
HS_{t-2}	$\phi_{12,2}$	-0.041	0.036	-0.107	0.001	$\phi_{22,2}$	0.000	0.006	-0.001	0.005	$\phi_{32,2}$	0.001	0.009	-0.001	0.028	$\phi_{42,2}$	-0.001	0.008	-0.020	0.001
TX_{t-2}	$\phi_{13,2}$	0.000	0.005	-0.001	0.001	$\phi_{23,2}$	0.001	0.008	-0.001	0.021	$\phi_{33,2}$	-0.006	0.018	-0.066	0.001	$\phi_{43,2}$	0.000	0.007	-0.001	0.001
NK_{t-2}	$\phi_{14,2}$	0.001	0.008	-0.001	0.025	$\phi_{24,2}$	0.001	0.005	-0.001	0.012	$\phi_{34,2}$	0.000	0.003	-0.001	0.000	$\phi_{44,2}$	-0.002	0.009	-0.036	0.001
	c_1	0.007	0.020	-0.001	0.072	c_2	-0.001	0.007	-0.008	0.001	c_3	-0.001	0.006	-0.011	0.001	c_4	0.001	0.009	-0.001	0.014
	$\sigma_{1,1}$	0.986	0.043	0.907	1.074	$\sigma_{1,2}$	0.279	0.034	0.214	0.348	$\sigma_{1,3}$	0.241	0.029	0.186	0.298	$\sigma_{1,4}$	0.242	0.040	0.165	0.321
$\mathbf{\Sigma}_{\varepsilon}$						$\sigma_{2,2}$	1.135	0.049	1.041	1.234	$\sigma_{2,3}$	0.502	0.033	0.438	0.570	$\sigma_{2,4}$	0.469	0.043	0.386	0.555
											$\sigma_{3,3}$	0.807	0.035	0.742	0.880	$\sigma_{3,4}$	0.434	0.037	0.363	0.508
																$\sigma_{4,4}$	1.553	0.068	1.428	1.694

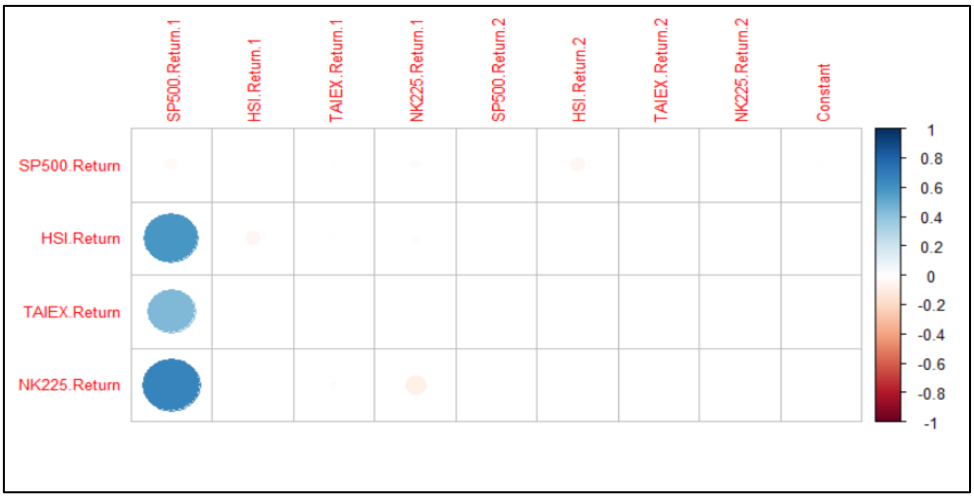


Figure 5: A Visualized Approach for the Parameter Estimates of the Best Model under $(c_0, c_1) = (0.01, 20)$ during Period II (from 2011 to 2015).

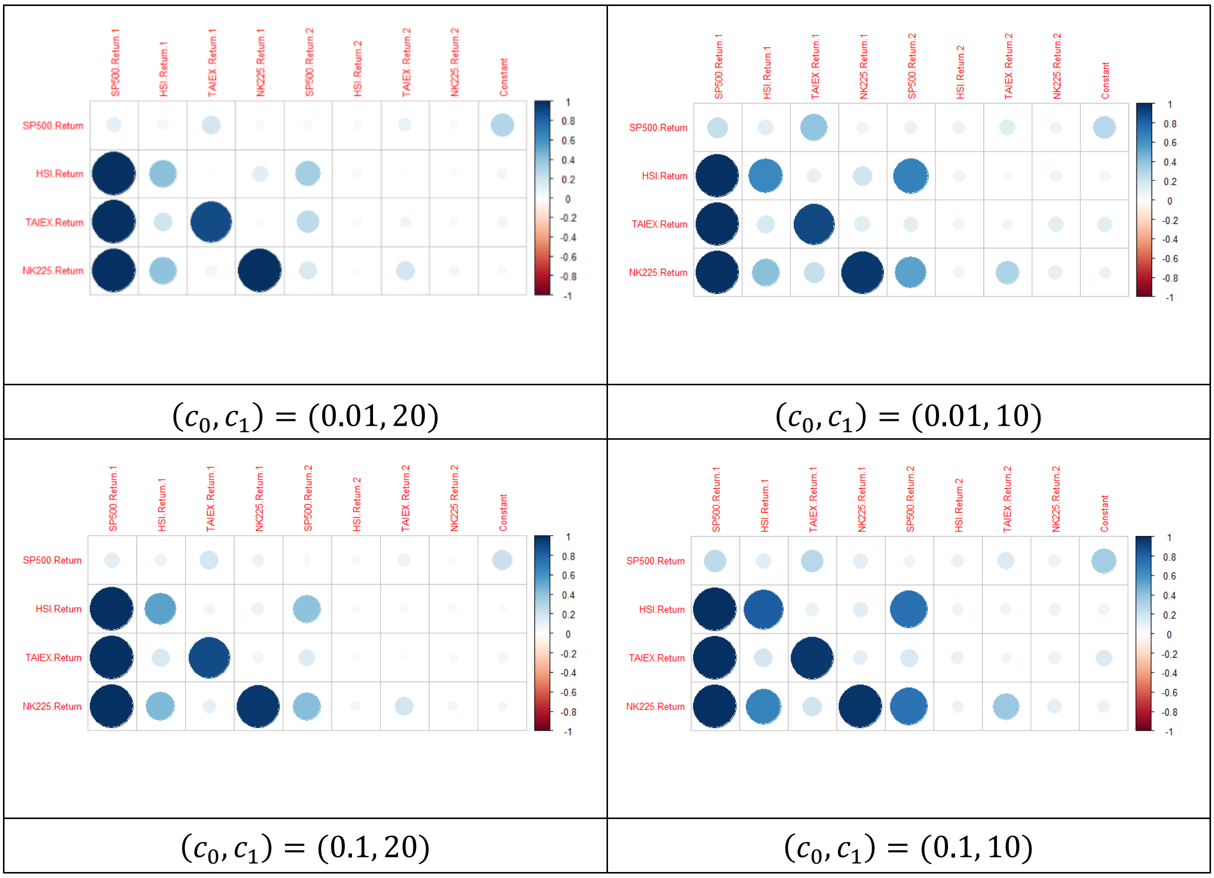
For the results of parameter estimations under $(c_0, c_1) = (0.01, 20)$ shown in Table 5, we obtain that the parameter estimates are estimated close to zero while the corresponding variables are unselected, and the selected variables are estimated apart from zero, especially for coefficients $\phi_{14,1}$, $\phi_{21,1}$, $\phi_{31,1}$, and $\phi_{41,1}$ that have significant estimates. The coefficients $\phi_{14,1}$, $\phi_{21,1}$, $\phi_{31,1}$, and $\phi_{41,1}$ have significant positive estimates. It means that the variables NK_{t-1} for S&P 500 and SP_{t-1} for HSI, TAIEX, and Nikkei 225 have positive influences. Thus, the parameter estimates of Table 5 are consistent with the results of the subsets' selection in Table 4. Figure 5 also simply points out the significant coefficients according to the above findings.

For Period III, the PIP's of most of variables vary continuously compared to the previous two periods. In this period, we obtain in Figure 6 that no significant PIP appears in the equation of S&P 500, while the variable SP_{t-1} is still important for HSI, TAIEX, and Nikkei 225 with high PIP's. The variables NK_{t-1} for Nikkei 225 and HS_{t-1} for HSI sustain respectively influence Nikkei 225 and HSI throughout the three periods. A notable variable, TX_{t-1} for TAIEX, appears with high PIP in Period III. For other variables, the variables HS_{t-1} for Nikkei 225 and SP_{t-2} for HSI are also possibly important in the VAR model of Period III.

For the subsets' selection, we focus on the results of $(c_0, c_1) = (0.01, 20)$ and $(0.1, 20)$ in Table 6. We find that the top 5 subsets under $(c_0, c_1) = (0.01, 20)$ and $(0.1, 20)$ have similar results for some digits of the code. The first two digits with numbers "3" and "5" or "4" are frequently observed in the selected subsets. It means that SP_{t-1} for HSI, TAIEX, and Nikkei 225 and HS_{t-1} for HSI are necessary for the VAR model. The third and fourth digits both have a number "2" obtained for a large number of selected subsets - that is, the variables HS_{t-1} for

Nikkei 225 and TX_{t-1} for TAIEX are considerably important in the model. Finally, we find the number "5" or "4" at the sixth digit, which presents that the variables NK_{t-1} for Nikkei 225 and SP_{t-2} for HSI may be important in the VAR model. Thus, we can potentially summarize eight variables, SP_{t-1} for HSI, TAIEX, and Nikkei 225, HS_{t-1} for HSI and Nikkei 225, TX_{t-1} for TAIEX, NK_{t-1} for Nikkei 225, and SP_{t-2} for HSI, as the important variables in the VAR model of Period III.

We finally show the parameter estimations under $(c_0, c_1) = (0.01, 20)$ in Table 7. We simply obtain that the coefficients $\phi_{21,1}$, $\phi_{31,1}$, and $\phi_{41,1}$ of SP_{t-1} for HSI, TAIEX, and Nikkei 225 are significantly estimated with a large positive value. It means that the variable SP_{t-1} has positive influences on HSI, TAIEX, and Nikkei 225 index returns. For other variables, we again find that the unselected variables are estimated close to zero, and the selected variables have estimated values that are far from zero. Observing from Figure 7, the figure again reflects the estimated values of parameters.



$(c_0, c_1) = (0.01, 20)$ | $(c_0, c_1) = (0.01, 10)$ | $(c_0, c_1) = (0.1, 20)$ | $(c_0, c_1) = (0.1, 10)$

Figure 6: A Visualized Approach for Presenting PIP's of Each Variable during Period III (from 2016 to 2019) under Four Combinations of (c_0, c_1).

Table 6: Results of Subsets' Selection for Period III (from 2016 to 2019).

(c_0, c_1)	$(\mathbf{0.1}, \mathbf{10})$	$(\mathbf{0.1}, \mathbf{20})$	$(\mathbf{0.01}, \mathbf{10})$	$(\mathbf{0.01}, \mathbf{20})$
Best	352205200000(0.57%) $\begin{bmatrix} 0&0&0&0&0&0&0&0&0 \\ 1&1&0&0&1&0&0&0&0 \\ 1&0&1&0&0&0&0&0&0 \\ 1&1&0&1&1&0&0&0&0 \end{bmatrix}$	352205200000(2.17%) $\begin{bmatrix} 0&0&0&0&0&0&0&0&0 \\ 1&1&0&0&1&0&0&0&0 \\ 1&0&1&0&0&0&0&0&0 \\ 1&1&0&1&1&0&0&0&0 \end{bmatrix}$	352205200000(0.97%) $\begin{bmatrix} 0&0&0&0&0&0&0&0&0 \\ 1&1&0&0&1&0&0&0&0 \\ 1&0&1&0&0&0&0&0&0 \\ 1&1&0&1&1&0&0&0&0 \end{bmatrix}$	352205000000(3.52%) $\begin{bmatrix} 0&0&0&0&0&0&0&0&0 \\ 1&1&0&0&1&0&0&0&0 \\ 1&0&1&0&0&0&0&0&0 \\ 1&1&0&1&0&0&0&0&0 \end{bmatrix}$
2nd Best	352205200010(0.53%) $\begin{bmatrix} 0&0&0&0&0&0&0&0&1 \\ 1&1&0&0&1&0&0&0&0 \\ 1&0&1&0&0&0&0&0&0 \\ 1&1&0&1&1&0&0&0&0 \end{bmatrix}$	340204000000(1.73%) $\begin{bmatrix} 0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0 \\ 1&0&1&0&0&0&0&0&0 \\ 1&0&0&1&0&0&0&0&0 \end{bmatrix}$	350305200400(0.85%) $\begin{bmatrix} 0&0&0&0&0&0&0&0&0 \\ 1&1&0&0&1&0&0&0&0 \\ 1&0&1&0&0&0&0&0&0 \\ 1&0&1&1&1&0&1&0&0 \end{bmatrix}$	352205200400(2.92%) $\begin{bmatrix} 0&0&0&0&0&0&0&0&0 \\ 1&1&0&0&1&0&0&0&0 \\ 1&0&1&0&0&0&0&0&0 \\ 1&1&0&1&1&0&1&0&0 \end{bmatrix}$
3rd Best	352205200410(0.47%) $\begin{bmatrix} 0&0&0&0&0&0&0&0&1 \\ 1&1&0&0&1&0&0&0&0 \\ 1&0&1&0&0&0&0&0&0 \\ 1&1&0&1&1&0&1&0&0 \end{bmatrix}$	352204000000(1.70%) $\begin{bmatrix} 0&0&0&0&0&0&0&0&0 \\ 1&1&0&0&0&0&0&0&0 \\ 1&0&1&0&0&0&0&0&0 \\ 1&1&0&1&0&0&0&0&0 \end{bmatrix}$	341225000000(0.77%) $\begin{bmatrix} 0&0&1&0&0&0&0&0&0 \\ 1&0&0&1&1&0&0&0&0 \\ 1&0&1&0&0&0&0&0&0 \\ 1&0&0&1&0&0&0&0&0 \end{bmatrix}$	345204000000(2.23%) $\begin{bmatrix} 0&0&1&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0 \\ 1&1&1&0&0&0&0&0&0 \\ 1&0&0&1&0&0&0&0&0 \end{bmatrix}$
4th Best	352205200400(0.39%) $\begin{bmatrix} 0&0&0&0&0&0&0&0&0 \\ 1&1&0&0&1&0&0&0&0 \\ 1&0&1&0&0&0&0&0&0 \\ 1&1&0&1&1&0&1&0&0 \end{bmatrix}$	352205200400(1.06%) $\begin{bmatrix} 0&0&0&0&0&0&0&0&0 \\ 1&1&0&0&1&0&0&0&0 \\ 1&0&1&0&0&0&0&0&0 \\ 1&1&0&1&1&0&1&0&0 \end{bmatrix}$	352301200410(0.64%) $\begin{bmatrix} 0&0&0&0&0&0&0&0&1 \\ 1&1&0&0&1&0&0&0&0 \\ 1&0&1&0&0&0&0&0&0 \\ 1&1&1&0&1&0&1&0&0 \end{bmatrix}$	340204000010(2.20%) $\begin{bmatrix} 0&0&0&0&0&0&0&0&1 \\ 1&0&0&0&0&0&0&0&0 \\ 1&0&1&0&0&0&0&0&0 \\ 1&0&0&1&0&0&0&0&0 \end{bmatrix}$
5th Best	752205200400(0.32%) $\begin{bmatrix} 1&0&0&0&0&0&0&0&0 \\ 1&1&0&0&1&0&0&0&0 \\ 1&0&1&0&0&0&0&0&0 \\ 1&1&0&1&1&0&1&0&0 \end{bmatrix}$	352204200000(0.93%) $\begin{bmatrix} 0&0&0&0&0&0&0&0&0 \\ 1&1&0&0&0&0&0&0&0 \\ 1&0&1&0&0&0&0&0&0 \\ 1&1&0&1&1&0&0&0&0 \end{bmatrix}$	751414000000(0.63%) $\begin{bmatrix} 1&0&1&0&0&0&0&0&0 \\ 1&1&1&0&0&0&0&0&0 \\ 1&0&0&1&0&0&0&0&0 \\ 1&0&0&1&0&0&0&0&0 \end{bmatrix}$	340204000000(1.97%) $\begin{bmatrix} 0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0 \\ 1&0&1&0&0&0&0&0&0 \\ 1&0&0&1&0&0&0&0&0 \end{bmatrix}$

Table 7: Results of Parameter Estimations of the Best Model under $(c_0, c_1) = (0.01, 20)$ for Period III (from 2016 to 2019).

Variable	S&P 500					HSI					TAIEX					Nikkei 225				
	Parameter	Mean	SD	95% C. I.		Parameter	Mean	SD	95% C. I.		Parameter	Mean	SD	95% C. I.		Parameter	Mean	SD	95% C. I.	
SP_{t-1}	$\phi_{11,1}$	-0.006	0.020	-0.080	0.001	$\phi_{21,1}$	**0.462**	**0.045**	**0.373**	**0.551**	$\phi_{31,1}$	**0.383**	**0.032**	**0.320**	**0.447**	$\phi_{41,1}$	**0.603**	**0.049**	**0.508**	**0.701**
HS_{t-1}	$\phi_{12,1}$	0.002	0.010	-0.001	0.041	$\phi_{22,1}$	-0.041	0.053	-0.151	0.001	$\phi_{32,1}$	0.010	0.024	-0.001	0.083	$\phi_{42,1}$	-0.039	0.054	-0.162	0.001
TX_{t-1}	$\phi_{13,1}$	0.013	0.031	-0.001	0.107	$\phi_{23,1}$	0.000	0.008	-0.001	0.001	$\phi_{33,1}$	-0.093	0.045	-0.170	0.000	$\phi_{43,1}$	-0.004	0.021	-0.081	0.001
NK_{t-1}	$\phi_{14,1}$	0.001	0.007	-0.001	0.009	$\phi_{24,1}$	-0.005	0.017	-0.064	0.001	$\phi_{34,1}$	0.000	0.005	-0.001	0.001	$\phi_{44,1}$	**-0.122**	**0.034**	**-0.187**	**-0.053**
SP_{t-2}	$\phi_{11,2}$	0.000	0.007	-0.001	0.001	$\phi_{21,2}$	0.032	0.051	-0.001	0.155	$\phi_{31,2}$	-0.015	0.029	-0.095	0.001	$\phi_{41,2}$	0.022	0.054	-0.001	0.182
HS_{t-2}	$\phi_{12,2}$	-0.001	0.007	-0.001	0.001	$\phi_{22,2}$	-0.001	0.007	-0.011	0.001	$\phi_{32,2}$	0.001	0.005	-0.001	0.002	$\phi_{42,2}$	-0.001	0.006	-0.001	0.001
TX_{t-2}	$\phi_{13,2}$	0.005	0.018	-0.001	0.068	$\phi_{23,2}$	0.000	0.005	-0.001	0.001	$\phi_{33,2}$	0.001	0.008	-0.001	0.021	$\phi_{43,2}$	-0.015	0.036	-0.128	0.001
NK_{t-2}	$\phi_{14,2}$	0.000	0.004	-0.001	0.001	$\phi_{24,2}$	0.000	0.005	-0.001	0.001	$\phi_{34,2}$	0.000	0.004	-0.001	0.005	$\phi_{44,2}$	-0.001	0.009	-0.024	0.001
	c_1	0.014	0.026	-0.001	0.084	c_2	0.000	0.006	-0.009	0.001	c_3	0.002	0.008	-0.001	0.028	c_4	-0.001	0.007	-0.009	0.001
	$\sigma_{1,1}$	0.621	0.030	0.564	0.683	$\sigma_{1,2}$	0.197	0.027	0.144	0.251	$\sigma_{1,3}$	0.125	0.020	0.086	0.165	$\sigma_{1,4}$	0.203	0.030	0.146	0.263
$\boldsymbol{\Sigma}_{\varepsilon}$						$\sigma_{2,2}$	0.962	0.047	0.873	1.059	$\sigma_{2,3}$	0.438	0.029	0.383	0.497	$\sigma_{2,4}$	0.507	0.041	0.431	0.591
											$\sigma_{3,3}$	0.526	0.026	0.478	0.580	$\sigma_{3,4}$	0.380	0.030	0.323	0.441
																$\sigma_{4,4}$	1.129	0.056	1.026	1.245

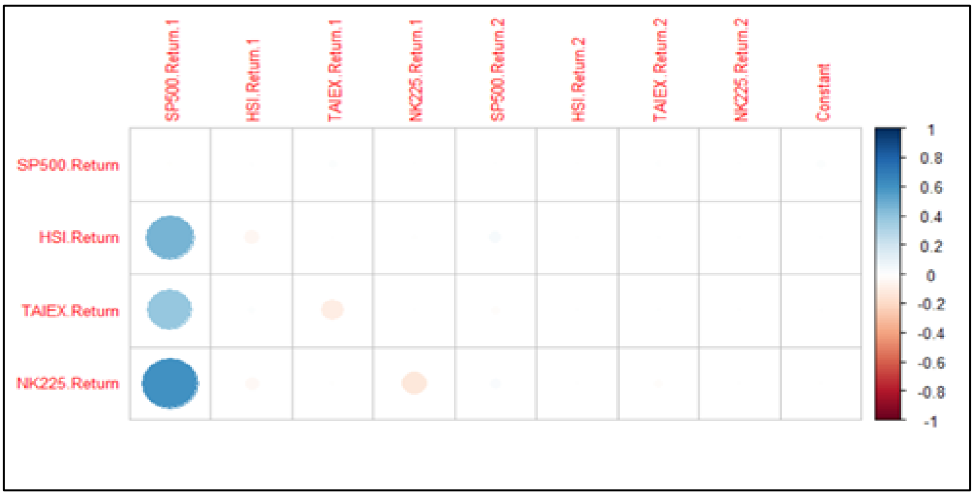


Figure 7: A Visualized Approach for the Parameter Estimates of the Best Model under $(c_0, c_1) = (0.01, 20)$ during Period III (from 2016 to 2019).

5. Conclusions

The unrestricted VAR model is recognized as presenting the problem of over-parameterization. This study thus employs a Bayesian variable selection method for VAR model restrictions to identify the dynamic relationships of financial time series. We apply the method for selection possible subsets of variables among four Pacific Rim stock market indices: S&P 500, HSI, TAIEX, and Nikkei 225. We find that the previous information of S&P 500 dominates the relationships of the four index returns and obtain an AR(1) effect of the endogenous variable in both HSI and Nikkei 225 index returns for Period I (from 2007 to 2010).

For Period II (from 2011 to 2015), the previous information of S&P 500 still plays an important role for HSI, TAIEX, and Nikkei 225, but the influence of S&P 500 itself decreases compared to Period I. The influences of NK_{t-1} for all returns and HS_{t-2} for S&P 500 are worth emphasizing, while the PIP's of NK_{t-1} for all returns and HS_{t-2} for S&P 500 significantly arise during Period II. Finally, in Period III the influences of SP_{t-1} for HSI, TAIEX, and Nikkei 225, HS_{t-1} for HSI, and NK_{t-1} for Nikkei 225 are significant with high PIP's. An AR(1) effect of TAIEX appears during Period III, while there is a high PIP of TX_{t-1} for TAIEX; otherwise, the influence of HS_{t-1} for Nikkei 225 is worth highlighting, while the PIP of HS_{t-1} for Nikkei 225 arises in this period.

This paper employs the method of Bayesian variable selection to estimate the parameters of VAR models and to identify the possible subsets of variables from a large number of candidates. We propose an alternative approach to inference the dynamic relationships of financial time series apart from the traditional inference on all the VAR coefficients. One can extend this technique to more complicated models such as nonlinear VAR models or VAR model with conditional heteroskedastic variances, which is worth investigating in future works.

Acknowledgement

The authors thank the Editor and two anonymous referees for their valuable time and careful comments to improve the quality of this paper. The research of Feng-Chi Liu is supported by the Ministry of Science and Technology of Taiwan grant MOST 107-2118-M-035-001-MY2.

Appendix

Table 8: Results of Parameter Estimations of the Best Model under $(c_0, c_1) = (0.01, 20)$ for the Three Periods without Variable Selection.

Period I: From January 5, 2007 to December 30, 2010.												
Variable		S&P 500			HSI			TAIEX			Nikkei 225	
	Parameter	Mean	SD	Parameter	Mean	SD	Parameter	Mean	SD	Parameter	Mean	SD
SP_{t-1}	$\phi_{11,1}$	**-0.136**	**0.034**	$\phi_{21,1}$	**0.517**	**0.041**	$\phi_{31,1}$	**0.336**	**0.029**	$\phi_{41,1}$	**0.559**	**0.034**
HS_{t-1}	$\phi_{12,1}$	0.000	0.000	$\phi_{22,1}$	**-0.188**	**0.025**	$\phi_{32,1}$	0.000	0.000	$\phi_{42,1}$	0.000	0.000
TX_{t-1}	$\phi_{13,1}$	0.000	0.000	$\phi_{23,1}$	0.000	0.000	$\phi_{33,1}$	0.000	0.000	$\phi_{43,1}$	0.000	0.000
NK_{t-1}	$\phi_{14,1}$	0.000	0.000	$\phi_{24,1}$	0.000	0.000	$\phi_{34,1}$	0.000	0.000	$\phi_{44,1}$	**-0.076**	**0.024**
SP_{t-2}	$\phi_{11,2}$	0.000	0.000	$\phi_{21,2}$	**0.145**	**0.030**	$\phi_{31,2}$	0.000	0.000	$\phi_{41,2}$	0.000	0.000
HS_{t-2}	$\phi_{12,2}$	0.000	0.000	$\phi_{22,2}$	0.000	0.000	$\phi_{32,2}$	0.000	0.000	$\phi_{42,2}$	0.000	0.000
TX_{t-2}	$\phi_{13,2}$	0.000	0.000	$\phi_{23,2}$	0.000	0.000	$\phi_{33,2}$	0.000	0.000	$\phi_{43,2}$	0.000	0.000
NK_{t-2}	$\phi_{14,2}$	0.000	0.000	$\phi_{24,2}$	0.000	0.000	$\phi_{34,2}$	0.000	0.000	$\phi_{44,2}$	0.000	0.000
	c_1	0.000	0.000	c_2	0.000	0.000	c_3	0.000	0.000	c_4	0.000	0.000
Period II: From January 5, 2011 to December 30, 2015.												
Variable		S&P 500			HSI			TAIEX			Nikkei 225	
	Parameter	Mean	SD	Parameter	Mean	SD	Parameter	Mean	SD	Parameter	Mean	SD
SP_{t-1}	$\phi_{11,1}$	0.000	0.000	$\phi_{21,1}$	**0.595**	**0.032**	$\phi_{31,1}$	**0.446**	**0.026**	$\phi_{41,1}$	**0.668**	**0.038**
HS_{t-1}	$\phi_{12,1}$	0.000	0.000	$\phi_{22,1}$	**-0.076**	**0.023**	$\phi_{32,1}$	0.000	0.000	$\phi_{42,1}$	0.000	0.000
TX_{t-1}	$\phi_{13,1}$	0.000	0.000	$\phi_{23,1}$	0.000	0.000	$\phi_{33,1}$	0.000	0.000	$\phi_{43,1}$	0.000	0.000
NK_{t-1}	$\phi_{14,1}$	0.000	0.000	$\phi_{24,1}$	0.000	0.000	$\phi_{34,1}$	0.000	0.000	$\phi_{44,1}$	**-0.092**	**0.025**
SP_{t-2}	$\phi_{11,2}$	0.000	0.000	$\phi_{21,2}$	0.000	0.000	$\phi_{31,2}$	0.000	0.000	$\phi_{41,2}$	0.000	0.000
HS_{t-2}	$\phi_{12,2}$	0.000	0.000	$\phi_{22,2}$	0.000	0.000	$\phi_{32,2}$	0.000	0.000	$\phi_{42,2}$	0.000	0.000
TX_{t-2}	$\phi_{13,2}$	0.000	0.000	$\phi_{23,2}$	0.000	0.000	$\phi_{33,2}$	0.000	0.000	$\phi_{43,2}$	0.000	0.000
NK_{t-2}	$\phi_{14,2}$	0.000	0.000	$\phi_{24,2}$	0.000	0.000	$\phi_{34,2}$	0.000	0.000	$\phi_{44,2}$	0.000	0.000
	c_1	0.000	0.000	c_2	0.000	0.000	c_3	0.000	0.000	c_4	0.000	0.000
Period III: From January 5, 2016 to December 30, 2019.												
Variable		S&P 500			HSI			TAIEX			Nikkei 225	
	Parameter	Mean	SD	Parameter	Mean	SD	Parameter	Mean	SD	Parameter	Mean	SD
SP_{t-1}	$\phi_{11,1}$	0.000	0.000	$\phi_{21,1}$	**0.484**	**0.042**	$\phi_{31,1}$	**0.392**	**0.031**	$\phi_{41,1}$	**0.611**	**0.046**
HS_{t-1}	$\phi_{12,1}$	0.000	0.000	$\phi_{22,1}$	**-0.110**	**0.029**	$\phi_{32,1}$	0.000	0.000	$\phi_{42,1}$	**-0.099**	**0.037**
TX_{t-1}	$\phi_{13,1}$	0.000	0.000	$\phi_{23,1}$	0.000	0.000	$\phi_{33,1}$	**-0.115**	**0.027**	$\phi_{43,1}$	0.000	0.000
NK_{t-1}	$\phi_{14,1}$	0.000	0.000	$\phi_{24,1}$	0.000	0.000	$\phi_{34,1}$	0.000	0.000	$\phi_{44,1}$	**-0.097**	**0.032**
SP_{t-2}	$\phi_{11,2}$	0.000	0.000	$\phi_{21,2}$	**0.088**	**0.035**	$\phi_{31,2}$	0.000	0.000	$\phi_{41,2}$	0.000	0.000
HS_{t-2}	$\phi_{12,2}$	0.000	0.000	$\phi_{22,2}$	0.000	0.000	$\phi_{32,2}$	0.000	0.000	$\phi_{42,2}$	0.000	0.000
TX_{t-2}	$\phi_{13,2}$	0.000	0.000	$\phi_{23,2}$	0.000	0.000	$\phi_{33,2}$	0.000	0.000	$\phi_{43,2}$	0.000	0.000
NK_{t-2}	$\phi_{14,2}$	0.000	0.000	$\phi_{24,2}$	0.000	0.000	$\phi_{34,2}$	0.000	0.000	$\phi_{44,2}$	0.000	0.000
	c_1	0.000	0.000	c_2	0.000	0.000	c_3	0.000	0.000	c_4	0.000	0.000

References

Campbell, J. Y., Lo. A. W., & MacKinlay. A. C. (1997). *The Econometrics of Financial Markets*. Princeton University Press, New Jersey.

Chen, C. W. S., Liu. F. C., & Gerlach. R. (2011). Bayesian Subset Selection for Threshold Autoregressive Moving-Average Models. *Computational Statistics*, **26**, 1-30.

Crespo Cuaresma, J., Feldkircher. M. & Huber. F. (2016). Forecasting with Global Vector Autoregressive Models: A Bayesian Approach. *Journal of Applied Econometrics*, **31**, 1371-1391.

Culbertson, K. (1996). *Quantitative Financial Economics: Stocks, Bonds and Foreign Exchange*. John Wiley and Sons, Chichester.

Davis, R. A., Zang. P., & Zheng. T. (2016). Sparse Vector Autoregressive Modeling. *Journal of Computational and Graphical Statistics*, **25**, 1077-1096.

Feldkircher, M. & Huber. F. (2016). The International Transmission of US Shocks-Evidence from Bayesian Global Vector Autoregressions. *European Economic Review*, **81**, 167-188.

Franz, X. M. (2019). *Stochastic Search Variable Selection*, Retrieved from: https://cran.r-project.org/web/packages/bvartools/vignettes/ssvs.html.

George, E. I. & McCulloch. R. E. (1993). Variable Selection via Gibbs Sampling. *Journal of the American Statistical Association*, **88**, 881-889.

George, E. I. & McCulloch. R. E. (1997). Approaches to Bayesian Variable Selection. *Statistica Sinica*, **7**, 339-379.

George, E. I., Sun. D., & Ni. S. (2008). Bayesian Stochastic Search for VAR Mode Restrictions. *Journal of Econometrics*, **142**, 553-580.

Hamilton, J. D. (1994). *Time Series Analysis*. Princeton University Press, Princeton.

Koop, G. (2013). Forecasting with Medium and Large Bayesian VARs. *Journal of Applied Econometrics*, 28, 177-203.

Korobilis, D. (2013). VAR Forecasting Using Bayesian Variable Selection. *Journal of Applied Econometrics*, 28, 204-230.

Koop, G. & Korobilis. D. (2010). *Bayesian Multivariate Time Series Methods for Empirical Macroeconomics*. Now Publishers.

Li, L., Leng. S., Yang. J., & Yu. M. (2016). Stock Market Autoregressive Dynamics: A Multinational Comparative Study with Quantile Regression. *Mathematical Problems in Engineering*. Online: https://doi.org/10.1155/2016/1285768.

Mills, T. C. (1999). *The Econometric Modeling of Financial Time Series, Second Edition*. Cambridge University Press, Cambridge.

Sims, C. A. (1980). Macroeconomics and Reality. *Econometrica*, **48**, 1-48.

So, M. K. P., Chen. C. W. S., & Liu. F. C. (2006). Best subset selection of autoregressive models with exogenous variables and generalized autoregressive conditional heteroscedasticity errors. *Journal of the Royal Statistical Society Series C*, **55**, 201-224.

Stock, J. H. & Watson. M. W. (2001). Vector Autoregressions. *Journal of Economic Perspectives*, **15**, 101-115.

Tsay, R. (2001). *Analysis of Financial Time Series*. John Wiley & Sons. New York.

Yu, K., Chen. C. W. S., Reed. C., & Dunson. D. B. (2013). Bayesian Variable Selection in Quantile Regression. *Statistics and Its Interface*, **6**, 261-274.