Asymmetric Volatility, Investor Sentiment, and Market Co-movements: An Analysis of Cryptocurrencies and U.S. Equities Using GARCH and LSTM Models

Ai-Chi Hsu

Finance, National Yunlin University of Science and Technology, Taiwan

Chia Hung Chen*

Finance, National Yunlin University of Science and Technology, Taiwan

Chiu-Ming Hsiao

Finance, National Yunlin University of Science and Technology, Taiwan

Abstract

This study examines the dynamic co-movement, return spillovers, and volatility transmission mechanisms between major cryptocurrencies and the U.S. equity market before, during, and after the COVID-19 pandemic. In this study, co-movement refers to the degree to which asset prices or returns exhibit synchronous or mutually dependent fluctuations over time, reflecting the extent of information integration, investor behavior linkage, and systemic risk propagation across markets. Focusing on Bitcoin, Ethereum, Litecoin, and Ripple, we analyze how their return and volatility dynamics interact with the S&P 500 Index and investor sentiment indicators across different pandemic phases.

The empirical findings reveal that, during the COVID-19 outbreak, return-based co-movement and spillovers among the four cryptocurrencies remain relatively weak, suggesting short-term market segmentation in return behavior. In contrast, volatility spillovers become substantially stronger, indicating heightened interdependence in risk transmission. These volatility spillovers originate from both own-market shocks and lagged cross-market influences, producing heterogeneous positive and negative effects on volatility across different assets. Moreover, the cryptocurrencies exhibit pronounced asymmetric behavior: negative shocks exert significantly greater effects on volatility than positive shocks.

In the post-pandemic period, negative news continues to dominate the volatility co-movement patterns of cryptocurrencies, reflecting their increased sensitivity to U.S. stock market fluctuations. Further spillover analysis shows that both the S&P 500 Index and the VIX influence cryptocurrency returns through volatility transmission channels. Specifically, the S&P 500 exerts a negative unidirectional spillover effect, whereas the VIX demonstrates either unidirectional or bidirectional transmission dynamics, highlighting the role of investor fear and sentiment in shaping cross-market linkages.

Keywords: COVID-19 Pandemic, Cryptocurrencies, Co-movement, VIX, Volatility Spillover

JEL Classifications: G17

*Corresponding author

E-mail: karl53345334@gmail.com

Address: 228 Liaoning Road, No. , Section 1, Beitun District,Taichung City.

1. Introduction

The World Health Organization (WHO) officially declared COVID-19 a global pandemic on March 11, 2020, an event that triggered unprecedented disruptions in global financial markets. As social distancing measures and lockdown policies significantly reduced in-person trading activities, investor interest increasingly shifted toward digital and decentralized financial assets, particularly cryptocurrencies. Despite their inherently high volatility, cryptocurrencies have generated substantial returns since the onset of the pandemic, reinforcing their growing relevance within modern financial systems.

Bitcoin (BTC), the largest and earliest cryptocurrency introduced in 2009, has experienced pronounced price fluctuations over time. Notably, in 2022, aggressive interest rate hikes by the U.S. Federal Reserve and the collapse of major cryptocurrency exchanges such as FTX led to a sharp decline in Bitcoin's market value. Compared with traditional financial assets, cryptocurrencies exhibit considerably higher volatility, making the analysis of market liquidity and risk transmission mechanisms especially important. As the pioneering cryptocurrency, Bitcoin has facilitated the rapid development of numerous alternative cryptocurrencies, most of which rely on blockchain technology. While prior research has largely focused on Bitcoin, a comprehensive understanding of cryptocurrency market volatility and its interaction with traditional and technology-driven financial assets remains essential.

Cryptocurrency prices are strongly influenced by global economic and political conditions. During the COVID-19 crisis, economic contraction and large-scale quantitative easing measures prompted some investors to perceive Bitcoin as a potential hedge against macroeconomic uncertainty. Empirical studies, such as Corbet et al. (2018), document strong interdependence within cryptocurrency markets and highlight their increasing importance in diversified investment portfolios. However, the cryptocurrency market also entails unique sources of risk, underscoring the necessity of examining its relationship with stock markets—particularly during periods of heightened uncertainty such as the COVID-19 pandemic.

Unlike conventional equity markets, cryptocurrency volatility appears to be more sensitive to epidemic-related shocks and uncertainty-driven factors. Consequently, understanding the volatility behavior of cryptocurrencies has become a central research issue. Key questions remain regarding whether instability in the U.S. stock market affects cryptocurrency performance and how investor sentiment and risk indicators shape return and volatility dynamics. Despite the growing body of literature on cryptocurrencies, relatively few studies have explicitly examined return and volatility spillovers during the COVID-19 period.

Existing research provides evidence of strong correlations, herd behavior, and co-movement within cryptocurrency markets. For instance, da Gama Silva et al. (2019) and Bouri et al. (2019) demonstrate that Bitcoin often plays a dominant role in driving market-wide co-movements. Song et al. (2019) identify clustering behavior among cryptocurrencies, while Bouri et al. (2018) propose dynamic linkages between stock prices and cryptocurrency trends. Moreover, Yousaf and Ali (2021) find that prior to COVID-19, spillovers between the U.S. stock market and cryptocurrencies were limited; however, during the pandemic, the S&P 500 exerted a significant influence on cryptocurrency returns. Similarly, Khan et al. (2023) document heightened volatility across global financial markets, including cryptocurrencies, during the pandemic. These findings underscore the importance of jointly analyzing cryptocurrency volatility and traditional financial markets.

Against this background, the present study investigates the co-movements of four major cryptocurrencies—Bitcoin (BTC), Ethereum (ETH), Litecoin (LTC), and Ripple (XRP)—across pre-pandemic, pandemic, and post-pandemic periods. Employing EGARCH, BEKK, and DCC-GARCH models, we examine return and volatility spillovers among these cryptocurrencies and their interactions with the S&P 500 index and the VIX, a widely used proxy for investor sentiment. In addition, we assess whether spillover effects exhibit unidirectional or bidirectional transmission patterns. The EGARCH results reveal pronounced asymmetric effects, indicating that negative shocks generate larger volatility responses than positive shocks. Following the COVID-19 period, the relationship between cryptocurrencies and the S&P 500 becomes increasingly positive, while correlations with the VIX turn more negative, suggesting that cryptocurrencies may serve as a hedge during periods of elevated uncertainty. Furthermore, dynamic volatility linkages identified through DCC-GARCH and long short-term memory (LSTM) deep learning models reveal positive correlations between the four cryptocurrencies and the S&P 500, alongside negative correlations with the VIX.

Prior studies, such as Baur and Dimpfl (2018), document asymmetric volatility effects in cryptocurrency markets that differ from those observed in stock markets, with positive shocks inducing larger volatility increases than negative shocks. Kumar and Ajaz (2019) and Qiao, Zhu, and Hau (2020) examine time-varying co-movement patterns among cryptocurrencies in terms of returns and volatility, while Hsu (2022) emphasizes that cryptocurrency volatility evolves over time in response to market conditions and risk events. Although existing research has largely focused on volatility characteristics and basic market structures, relatively limited attention has been devoted to co-movement dynamics. Our study contributes to the literature by providing a comprehensive analysis of co-movements among cryptocurrencies.

In addition, prior evidence regarding volatility spillovers between cryptocurrencies and stock markets remains mixed. Corbet et al. (2018) report strong spillover effects from the S&P 500 to cryptocurrencies, whereas Kurka (2019) and Zeng et al. (2020) find limited correlations between cryptocurrencies and traditional financial assets. These divergent findings highlight the need for further investigation into cross-market volatility transmission mechanisms. Given that many pre-COVID-19 studies may not fully capture the distinctive volatility dynamics observed during the pandemic, this study seeks to address this gap.

Recent research underscores the importance of pandemic-specific analysis. Conlon et al. (2020) find that Ether functioned as an effective hedge during COVID-19, while Özdemir (2022) reports significant volatility and spillover effects in BTC, ETH, and LTC markets during this period. As investor interest in cryptocurrencies continues to grow, understanding their volatility behavior and interdependencies with traditional financial markets has become increasingly critical. This study contributes to the literature by offering new empirical evidence on cryptocurrency co-movements and cross-market spillovers under pandemic-related uncertainty.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature and formulates the research questions. Section 3 describes the methodology. Section 4 presents and discusses the empirical results. Section 5 provides additional analyses, and Section 6 concludes the study.

2. Literature Review and Research Question

2.1 Co-movement and Volatility Spillovers Among Cryptocurrencies

Diebold and Yilmaz (2009) document a gradual upward trend in return spillovers using vector autoregression (VAR) models, while volatility spillovers are characterized by sharp, abrupt surges without a stable long-term pattern. Their subsequent work (Diebold and Yilmaz, 2012) introduces a generalized VAR-based spillover index to analyze daily volatility transmissions across stock, bond, foreign exchange, and commodity markets. They show that cross-market spillovers were already substantial prior to the 2007 global financial crisis but intensified dramatically during the crisis, especially following the collapse of Lehman Brothers, when spillovers from equity markets to other financial markets became markedly stronger.

Within the cryptocurrency domain, Dyhrberg (2016) employs an asymmetric GARCH model to examine Bitcoin's hedging capabilities, finding that BTC can effectively hedge against stock market fluctuations—such as those of the FTSE index—and short-term U.S. dollar exposure, exhibiting characteristics similar to gold. Baur and Dimpfl (2018) further highlight the unique volatility dynamics of cryptocurrencies, showing that "fear of missing out" behavior amplifies volatility following positive shocks. They report significant asymmetric volatility among the top 20 cryptocurrencies, where positive shocks induce disproportionately larger volatility increases compared with negative shocks—an effect less prevalent in traditional equity markets. Koutmos (2018) also finds that Bitcoin plays a leading role in return and volatility spillovers among major cryptocurrencies, with spillover intensities increasing over time, suggesting rising interdependence and contagion risk in the cryptocurrency ecosystem.

Kumar and Ajaz (2019) use wavelet coherence to uncover strong, time-varying co-movement patterns among Bitcoin, Ethereum, Litecoin, and Dash, with Bitcoin consistently leading market dynamics. They show that Bitcoin price increases stimulate market-wide demand and price appreciation among other cryptocurrencies, while Bitcoin price declines spread rapidly across the market. Urquhart and Zhang (2019) affirm Bitcoin's hedging properties, whereas Qiao, Zhu, and Hau

(2020) find that Bitcoin's returns and volatility exert significant and increasing influences on other major cryptocurrencies. Özdemir (2022) adds evidence of substantial volatility spillovers and interdependence among Bitcoin, Ethereum, and Litecoin, particularly during the second COVID-19 lockdown in late 2020. During the pandemic, the increased inflow of investors seeking higher returns intensified speculative behavior, herd effects, and volatility spillovers, thereby amplifying contagion risks within the cryptocurrency market.

2.2 Co-movement Between Cryptocurrencies, the Stock Market, and Investor Sentiment

Bouri et al. (2018) identify dynamic linkages between stock prices and cryptocurrency trends, emphasizing the interconnectedness of financial markets. Corbet et al. (2018) document significant return and volatility spillovers from the S&P 500 to cryptocurrencies, indicating that short-term diversification benefits may exist for investors. Guesmi et al. (2019) show strong return and volatility transmissions between Bitcoin and traditional assets such as gold, oil, and equities, whereas Kurka (2019) finds Bitcoin largely isolated from these markets in terms of shock propagation.

Liu and Serletis (2019), using a BEKK-GARCH framework, demonstrate that S&P 500 returns exert a positive effect on cryptocurrency returns, although volatility spillovers remain statistically insignificant. Tiwari, Raheem, and Kang (2019), employing a copula-ADCC-EGARCH approach, find low but time-varying correlations between six cryptocurrencies and the S&P 500, with cryptocurrencies—particularly Litecoin—serving as effective hedges against S&P 500 risk. Both markets exhibit stronger responses to negative shocks than to positive ones. Additionally, Bouri et al. (2020) use a smooth transition VAR-GARCH model and find Bitcoin to display substantially higher volatility than traditional financial assets. Consistent with this, Zeng et al. (2020) detect minimal spillovers from the S&P 500 to Bitcoin, suggesting weak integration between the two markets. Hsu (2022) reports increasing co-movement spillovers between cryptocurrencies (e.g., Bitcoin and Ethereum) and major currencies, driven by evolving market conditions and risk events.

Ghorbel and Jeribi (2021), applying a BEKK-GARCH model, show strong volatility spillovers among cryptocurrencies but weaker spillover relationships between cryptocurrencies and traditional assets such as U.S. equity indices, oil, and gold. However, during the early stages of the COVID-19 pandemic, correlations between cryptocurrencies, U.S. stock indices, and oil increased, signaling contagion effects under conditions of heightened uncertainty. Yousaf and Ali (2021) find negligible return and volatility spillovers between the U.S. stock market and cryptocurrencies prior to COVID-19. Yet during the pandemic, the S&P 500 exerted a unidirectional influence on cryptocurrency returns and volatility—especially in the case of Litecoin.

2.3 Research Questions

This study examines both return and volatility spillover effects, structured into two analytical components. Return spillovers are commonly investigated through Granger causality tests and VAR models and are influenced by a market's intrinsic characteristics as well as external transmissions from other markets. Volatility spillovers, in contrast, capture how fluctuations in one market propagate—positively or negatively—to others. The primary objective of this research is to analyze the co-movement and volatility dynamics of four major cryptocurrencies—Bitcoin (BTC), Ethereum (ETH), Litecoin (LTC), and Ripple (XRP)—across three distinct periods: pre-COVID-19, during the pandemic, and post-pandemic. Specifically, the study examines return–volatility interactions, identifies co-movement patterns among cryptocurrencies, determines the presence of volatility spillovers within the cryptocurrency market, evaluates causal relationships, and distinguishes between unidirectional and bidirectional spillover effects.

Previous studies offer important reference points. Aslanidis et al. (2019) examine conditional correlations among Bitcoin, Monero, Dash, and Ripple, and explore their linkages with the S&P 500 index, government bonds, and gold. Tiwari et al. (2019) analyze time-varying correlations between the S&P 500 and cryptocurrencies, highlighting their potential as hedging instruments. Ghorbel and Jeribi (2021) demonstrate—through a BEKK-GARCH model—that volatility spillovers are stronger among cryptocurrencies than between cryptocurrencies and traditional assets, and that correlations between cryptocurrencies, U.S. stock indices, and oil intensified during early 2020, reflecting contagion effects triggered by the COVID-19 pandemic.

Although Bitcoin has dominated scholarly attention, other major cryptocurrencies have been comparatively understudied. Recognizing the VIX as a critical measure of investor sentiment, the present study explores dynamic conditional relationships among BTC, ETH, LTC, XRP, the S&P 500 index, and the VIX. This analysis aims to determine the direction of spillovers (unidirectional vs. bidirectional), assess their magnitude, and provide deeper insights into cross-market interdependencies.

In summary, this study offers a comprehensive analysis of volatility co-movements and spillover effects among four leading cryptocurrencies, the S&P 500 index, and the VIX. By examining these interconnections across distinct pandemic-related phases, the study contributes to a deeper understanding of risk transmission and market dynamics in cryptocurrency and traditional financial markets.

3. Research Design

3.1 Vector autoregression (VAR)

VAR is a modeling technique that considers lagged variables, both from within themselves and from other variables. In this paper, we follow Diebold and Yilmaz's (2009) methodology to build a six-variable VAR model for log-transformed cryptocurrency prices.

$$Z_t = HZ_{t-1} + E. \tag{1}$$

$$Z_t = \begin{pmatrix} CB_t \\ CE_t \\ CL_t \\ CX_t \\ SP500_t \\ VIX_t \end{pmatrix} H = \begin{pmatrix} a_1 a_2 a_3 a_4 a_5 a_6 \\ b_1 b_2 b_3 b_4 b_5 b_6 \\ c_1 c_2 c_3 c_4 c_5 c_6 \\ d_1 d_2 d_3 d_4 d_5 d_6 \\ e_1 e_2 e_3 e_4 e_5 e_6 \\ f_1 f_2 f_3 f_4 f_5 f_6 \end{pmatrix} \mathrm{E} = \begin{pmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \\ e_{4t} \\ e_{5t} \\ e_{6t} \end{pmatrix} \tag{2}$$

, where CB_t= return of Bitcoin in time t, CE_t= return of Ethereum in time t, CL_t = return of Litecoin in time t, CX_t= return of Ripple in time t, $SP500_t$= return of the S&P500 index in time t, VIX_t= volatility of the investor sentiment index VIX in time t, e_t= residual term. Moreover, cryptocurrency return are calculated based on cryptocurrency market prices, denoted as Pt. The daily price return is computed as the difference between Pt and Pt-1, where Pt represents the daily closing price at time t, and Pt-1 represents the daily closing price at time t-1. Subsequently, the cryptocurrency return at time t is calculated as follows:

R_t = log (P_t/P_{t-1}) = logP_t - logP_{t-1}, where R_t= CB_t, CE_t, CL_t, CX_t, and they represent return of BTC, ETH, LTC, and XRP, respectively.

3.2 GARCH model

Engle (1982) introduces the Autoregressive Conditional Heteroskedasticity (ARCH) model, suggesting that error term variances are linked to previous error term variances, similar to the behavior of error terms in a standard Autoregressive (AR) process. Bollerslev (1986) extends this concept to propose the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model. The GARCH model is a valuable tool for modeling time series data with heteroscedastic errors, utilizing an autoregressive moving average process. It introduces conditional heteroscedasticity while preserving a homoscedastic unconditional error variance. It assumes that variance changes are linked to past error realizations and that these changes are temporary and random, deviating from a constant unconditional variance. Therefore, the GARCH model leverages historical time series volatility data to accommodate the time-varying nature of volatility. Through the integration of past volatility information, the GARCH model can forecast future volatility and evaluate the risk associated with financial assets.

This paper explores the link between return volatility in the S&P 500 index, investor sentiment (VIX), and four cryptocurrencies: BTC, ETH, LTC, and XRP. The GARCH model includes two equations: the mean equation and the variance equation. The mean equation describes the mean level of the time series and the variance equation describes the changes in volatility and is a function of past error terms and volatility. Model (3) is employed to investigate whether the volatility in the S&P 500 stock price index and the VIX index has a spillover impact on the conditional variances of these four cryptocurrencies. The mean equation for the i-th cryptocurrency is as follows:

$$R_{it} = c_{0i} + c_{1i} R_{it-1} + c_{2i} SP500_{t-1} + c_{3i} VIX_{t-1} + \varepsilon_{it} \text{ , } (\varepsilon_t | I_{t-1}) \sim N(0, \sigma^2) \tag{3}$$

, where i denotes the i-th cryptocurrency, which can be BTC, ETH, LTC, or XRP. R_{it} and R_{it-1} represent the returns of the i-th cryptocurrency in period t and period t-1, respectively. c_{0i} is the intercept. S&P 500 represents the return of the S&P 500. VIX is the volatility of the VIX fear index. ε_{t-1} represents the shock term of the return of the i-th cryptocurrency, which follows a normal distribution with mean 0 and variance σ_{it}^2. I_{t-1} represents the set of all information available before period t-1. The variance equation of the i-th cryptocurrency is as follows:

$$\sigma_{it}^2 = \omega_i + \sum_{k=1}^{p} \alpha_{ji}\varepsilon_{it-j}^2 + \sum_{k=1}^{q} \beta_{ki}\sigma_{it-k}^2 + \varphi_{1i}SP_{t-1}^2 + \varphi_{2i}VIX_{t-1}^2 \quad (4)$$

, where i denotes the i-th cryptocurrency, which can be BTC, ETH, LTC, or XRP. σ_{it}^2 and σ_{it-k}^2 represent the conditional variance of the i-th cryptocurrency at period t and the previous k periods, respectively, which describe the volatility of cryptocurrency. ε_{it-j}^2 represents the squared residual from the mean equation of the i-th cryptocurrency at lag j period, capturing the volatility in the cryptocurrency. α_{ji} is used to estimate the past impact of shocks on the i-th cryptocurrency at period j, that is, the impact of the cryptocurrency on its volatility when it encounters external influencing factors. β_{ki} is used to detect the historical volatility effect of the i-th cryptocurrency at period k, that is, the historical impact of the cryptocurrency's own volatility. j and k represent the past ARCH terms and past GARCH terms, respectively. They capture the heterogeneity and persistence of the cryptocurrency's volatility change process. p and q denotes the lag periods, p measures the impact of historical volatility on the current volatility and q measures the impact of the historical volatility residual on the current volatility. $\sum(\alpha_j + \beta_j)$ indicates the speed of convergence towards a long-term stable equilibrium and the persistence of volatility following shocks. A higher value suggests a slower convergence rate and a more extended period of volatility. φ_{1i} and φ_{2i} are utilized to evaluate the spillover effects of S&P 500 volatility and VIX volatility on the conditional variance of the four cryptocurrencies.

3.3 EGARCH model

We use the exponential GARCH model, introduced by Nelson (1991) and often referred to as the exponential generalized autoregressive conditionally heteroscedasticity (EGARCH) model. It is utilized to assess the impact of shocks on the four cryptocurrencies: BTC, ETH, LTC, and XRP. The EGARCH model is effective in determining whether volatility exhibits asymmetry. We employ the EGARCH model to examine the asymmetric impact of upward and downward movements. The variance equation for the i-th cryptocurrency in the EGARCH (*p, q*) model is as follows:

$$\log\sigma_{it}^2 = \omega_i + \sum_{j=1}^{p} \alpha_{ji}\left|\frac{\varepsilon_{it-j}}{\sigma_{it-j}}\right| + \sum_{k=1}^{q} \beta_{ki}\log\sigma_{it-k}^2 + \sum_{j=1}^{r} \gamma_{ji}\frac{\varepsilon_{it-j}}{\sigma_{it-j}} + \varphi_{1i}SP_{t-1}^2 + \varphi_{2i}VIX_{t-1}^2 \quad (5)$$

, where i represents the i-th cryptocurrency, which could be BTC, ETH, LTC, or XRP. $\log\sigma_{it}^2$ and $\log\sigma_{it-k}^2$ represent the logarithms of the conditional variances of the i-th cryptocurrency in period t and the previous k periods, respectively. α_{ji} is used to detect the shock effect of the i-th cryptocurrency in period j. β_{ki} is used to detect the volatility effect of the i-th cryptocurrency in period k. The relative relationship between ε_{it-j} and σ_{it-j} is used to detect the influence of information on conditional variance volatility. When $\gamma_{ji}=0$, it indicates that the volatility is symmetric. When $\gamma_{ji}<0$ and $\sum(\alpha_{ji}-\gamma_{ji})$ is greater than $\sum(\alpha_{ji}+\gamma_{ji})$, the volatility impact effect in the market will be that bad news has a greater impact effect than good news.

φ_{1i} and φ_{2i} are used to test whether the volatility of SP 500 and VIX has a spillover effect on the conditional variance of the i-th cryptocurrency, which can be BTC, ETH, LTC, or XRP. When $\frac{\varepsilon_{it-j}}{\sigma_{it-j}}$ is positive, the conditional variance coefficient is $\sum(\alpha_{ji}+\gamma_{ji})$ and the previous shock is positive. If it is negative, the conditional variance coefficient is $\sum(\alpha_{ji}-\gamma_{ji})$ and the previous shock is negative.

3.4 Multivariate GARCH

3.4.1 BEKK model

Engle and Kroner (1995) propose the asymmetric BEKK model, which is a type of multivariate GARCH model. This model estimates time-varying variances, which are not constant. We apply Engle and Kroner's (1995) model to analyze the transmission of volatility spillover effects among four cryptocurrencies, the SP500, and VIX indices. Furthermore, we explore whether these spillover effects occur unidirectionally or bidirectionally. The equation for the conditional covariance matrix in the BEKK model is as follows:

$$H_t = CC' + A'(\varepsilon_{t-1}\varepsilon'_{t-1})A + B'H_{t-1}B \tag{6}$$

, where $H_t=\begin{bmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{bmatrix}$ is conditional variance-covariance matrix, $h_{12,t}$ and $h_{21,t}$ are conditional covariances between two cryptocurrencies. $C=\begin{bmatrix} c_{11} & 0 \\ c_{21} & c_{22} \end{bmatrix}$ is the lower triangular matrix. $A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is a parameter matrix to capture ARCH effects. It reflects the impact of short-term shocks across different markets and the direction of volatility transmission concerning the variable itself (a_{11} and a_{22}) as well as the interaction between cryptocurrencies and other markets, including SP500 stock price index and VIX index (a_{12} and a_{21}). $B=\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ is a parameter matrix that represents GARCH effects. (b_{11} and b_{22}) are used to capture the long-term volatility of the variable itself; (b_{12} and b_{21}) represent the spillover and transmission direction of return volatility between cryptocurrencies and other markets (SP500 index and VIX index). ε_t denotes the error term between two conditional variables.

3.4.2 DCC-GARCH model

We further use Engle's (2002) dynamic conditional correlation GARCH (DCC-GARCH) model to examine the time-varying volatilities and correlations between the four cryptocurrencies and the SP500 stock index and VIX index. In the DCC model, the correlation coefficients can change over time based on its GARCH functional form. The conditional covariance matrix equation for the N-variable DCC-GARCH model is as follows:

$$H_t = D_t R_t D_t = (\rho_{ij}\sqrt{h_{iit}}\sqrt{h_{jjt}}) \tag{7}$$

, where $D_t = \begin{bmatrix} \sqrt{h_{11t}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sqrt{h_{NNt}} \end{bmatrix}$, $Q_t^* = \begin{bmatrix} \sqrt{q_{11t}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sqrt{q_{NNt}} \end{bmatrix}$. $D_t = diag\{h_{11t}^{1/2} \ldots\ldots h_{NNt}^{1/2}\}$ is a diagonal matrix of time-varying. $R_t = (\rho_{ij})$ is a matrix containing conditional correlation coefficient of ρ_{ij}. $R_t = Q_t^{*-1} Q_t Q_t^{*-1} = diag\,(q_{11,t}^{-1/2} \ldots . q_{NN,t}^{-1/2})\, Q_t diag(q_{11,t}^{-1/2} \ldots . q_{NN,t}^{-1/2}) \circ Q_t^*$ is D_t matrix for each conditional variance in the GARCH (1,1) model.

$h_{iit} = \omega_i + \alpha_i \varepsilon_{i,t-j}^2 + \beta h_{ii,t-1}$ i=1,...,N, where h_{iit} is a conditional variable and $\sqrt{h_{iit}}$ is the standard deviation of the conditional variable. N x N is symmetric positive definite matrix. $Q_t = (q_{ij,t})$ is represented as follows: $Q_t = (1-\alpha-\beta)\bar{Q} + \alpha\varepsilon_{t-1}\varepsilon_{t-1}' + \beta Q_{t-1}$, where Q_t and $\bar{Q}$ are N x N time-varying unconditional variables correlation matrix of ε_t. ε_t is a vector of errors. α and β are non-negative scalar parameters representing the shock effect of prior period's DCCs, and they satisfy the condition α+ β<1. Furthermore, the estimation model for dynamic correlations between conditional variables is given by the following formula:

$$\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t}q_{jj,t}}} \tag{8}$$

, where ρ_{ijt} represents the dynamic conditional correlation between the conditional variables, where i denotes four cryptocurrencies: BTC, ETH, LTC, and XRP, and J denotes the SP500 stock index and VIX index.

3.5 Data Description

This study employs six key variables: the daily closing prices of four major cryptocurrencies—Bitcoin (BTC), Ethereum (ETH), Litecoin (LTC), and Ripple (XRP)—along with the S&P 500 stock index and the VIX index. The sample period spans January 1, 2018, to December 31, 2022, and is divided into four distinct phases:

1. Full sample period (2018–2022)
2. Pre-COVID-19 period: January 1, 2018 to January 29, 2020
3. COVID-19 pandemic period: January 30, 2020 to February 23, 2022

4. Post-COVID-19 period: February 24, 2022 to December 31, 2022

The World Health Organization (WHO) officially confirmed the first case of COVID-19 on January 30, 2020; therefore, this date is adopted as the starting point of the pandemic period. After more than two years, the global health situation gradually improved. However, the onset of the Russia–Ukraine war on February 24, 2022, introduced new global economic turbulence. This geopolitical crisis triggered sharp increases in the prices of raw materials, energy, and crude oil, contributing to a prolonged inflationary environment. During 2022, the U.S. Federal Reserve implemented seven consecutive interest rate hikes totaling 175 basis points, raising the policy rate to 4.25%–4.50%. Accordingly, February 24, 2022 is designated as the beginning of the post-COVID-19 period in this study.

For empirical analysis, all daily closing prices are transformed using natural logarithms. After aligning the series based on common trading days and excluding non-trading days, the full sample (Phase 1) comprises 1,259 daily observations. The pre-COVID-19 period (Phase 2) and the COVID-19 pandemic period (Phase 3) each contain 522 observations, while the post-COVID-19 period (Phase 4) consists of 215 observations. Detailed information on data segmentation and descriptive statistics is provided in Table 1.

Table 1. Sample analysis

	Phase 1	Phase 2	Phase 3	Phase 4
	Full sample	Pre-COVID-19 pandemic	COVID-19 pandemic period	Post-COVID-19 pandemic
Period	January 1 2018 to December 31 2022	January 1 2018 to January 29 2020	January 30 2020 to February 23 2022	February 24 2022 to December 31 2022
Variables	Total sample	Total sample	Total sample	Total sample
BTC	1259	522	522	215
ETH	1259	522	522	215
LTC	1259	522	522	215
XRP	1259	522	522	215
SP500	1259	522	522	215
VIX	1259	522	522	215

The data used in this study were obtained from the following sources. Daily cryptocurrency prices for BTC, ETH, LTC, and XRP were collected from Yahoo Finance (https://finance.yahoo.com/). Data for the S&P 500 index were also retrieved from Yahoo Finance. As one of the most widely used indicators of the U.S. equity market, the S&P 500 reflects broad macroeconomic conditions due to its diversified composition across sectors and industries. The VIX index, sourced from the same provider, measures the market's expectation of future volatility and is widely interpreted as a proxy for investor sentiment and market risk. Initially introduced as the

"Volatility Index," the VIX is constructed based on the expected variance derived from S&P 500 index options.

4. Empirical Results

4.1 Descriptive statistics

This study analyzes the price dynamics of four major cryptocurrencies—Bitcoin (BTC), Ethereum (ETH), Litecoin (LTC), and Ripple (XRP)—together with the S&P 500 index and the VIX index over the period 2018–2022. The price trajectories presented in Figure 1 reveal a pronounced surge in cryptocurrency prices during 2020, coinciding with the outbreak of the COVID-19 pandemic. In particular, Bitcoin experienced a remarkable price increase of approximately 416% from the beginning of the year, reaching an all-time high of USD 68,991. However, following its peak in 2021, Bitcoin prices declined substantially amid growing global economic uncertainty, rising inflation, geopolitical tensions, and shifts in U.S. monetary policy. In 2022, the cryptocurrency market underwent severe turbulence following the collapse of the FTX exchange, leading to a rapid and pronounced price correction within a short period. Heightened investor concerns regarding the long-term sustainability of the cryptocurrency market further contributed to increased price volatility.

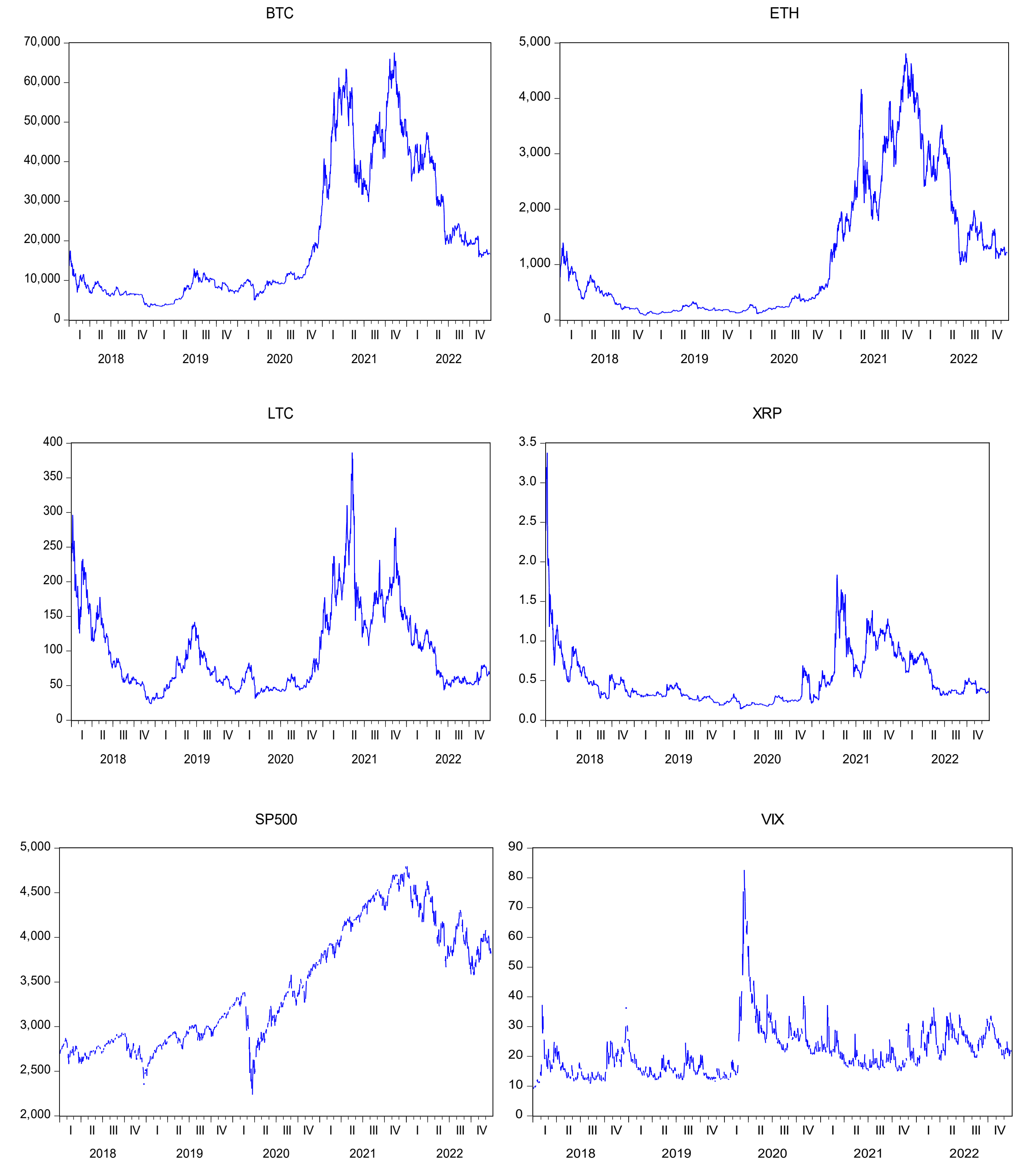

Figure 1. Price trends of four cryptocurrencies, SP500, and VIX from 2018 to 2022

Table 2 reports the descriptive statistics for the four cryptocurrencies and the two financial market indices. The results indicate that the mean returns of BTC and ETH are positive, whereas LTC and XRP exhibit negative average returns over the sample period. Among the cryptocurrencies, ETH records the highest average return at 0.0369%, while XRP displays the lowest at −0.139%. Both the S&P 500 index and the VIX index also show positive average returns.

In terms of return volatility, as measured by the standard deviation, the VIX index exhibits the highest value at 8.491%, reflecting substantial sensitivity to market shocks and uncertainty-related news. Among the four cryptocurrencies, BTC displays the lowest standard deviation at 4.657%, suggesting relatively lower risk compared with ETH, LTC, and XRP, all of which exhibit standard deviations exceeding 5%. Across all six variables, the S&P 500 index demonstrates the lowest volatility, with a standard deviation of 1.383%. This comparatively low level of volatility can be attributed to the diversified structure of the S&P 500, which tracks the performance of approximately 500 U.S. firms across 11 major industries. Such broad diversification reduces exposure to idiosyncratic risk and results in a narrower range of price fluctuations relative to the other variables examined in this study.

Table 2. Descriptive statistics

	BTC	ETH	LTC	XRP	SP500	VIX
Mean	0.0001	0.0004	-0.0010	-0.0014	0.0003	0.0005
Median	0.0009	0.0004	0.0008	-0.0014	0.0009	-0.0100
Maximum	0.2030	0.3435	0.2906	0.6267	0.0897	0.7682
Minimum	-0.4647	-0.5507	-0.4491	-0.5505	-0.1277	-0.2662
Std. Dev.	0.0466	0.0614	0.0612	0.0715	0.0138	0.0849
p50	0.0009	0.0004	0.0008	-0.0014	0.0009	-0.0100
p25	-0.0192	-0.0262	-0.0285	-0.0284	-0.0053	-0.0484
p75	0.0220	0.0309	0.0285	0.0249	0.0071	0.0358
Skewness	-1.0917	-0.7537	-0.6410	0.0294	-0.7918	1.5361
Kurtosis	13.4679	11.1251	9.3125	17.3718	15.8976	11.1112
Jarque-Bera	5998.3	3582.4	2176.6	10835.3	8857.9	3946.4
Probability	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Observations	1259	1259	1259	1259	1259	1259

The distributional properties further reveal that XRP and the VIX index exhibit right-skewed return distributions (skewness > 0), whereas the remaining four variables display left-skewed distributions (skewness < 0). All six variables are characterized by leptokurtic behavior, with kurtosis values exceeding 3, indicating pronounced peakness and fat tails in their return distributions. These features suggest a higher probability of extreme observations than would be expected under a normal distribution. Consistent with this observation, the Jarque–Bera test strongly rejects the null hypothesis of normality for all variables. Figure 2, which presents the return histograms for the four cryptocurrencies, the S&P 500, and the VIX index over the full sample period, visually confirms these characteristics, highlighting the presence of skewness, fat tails, and high peaks.

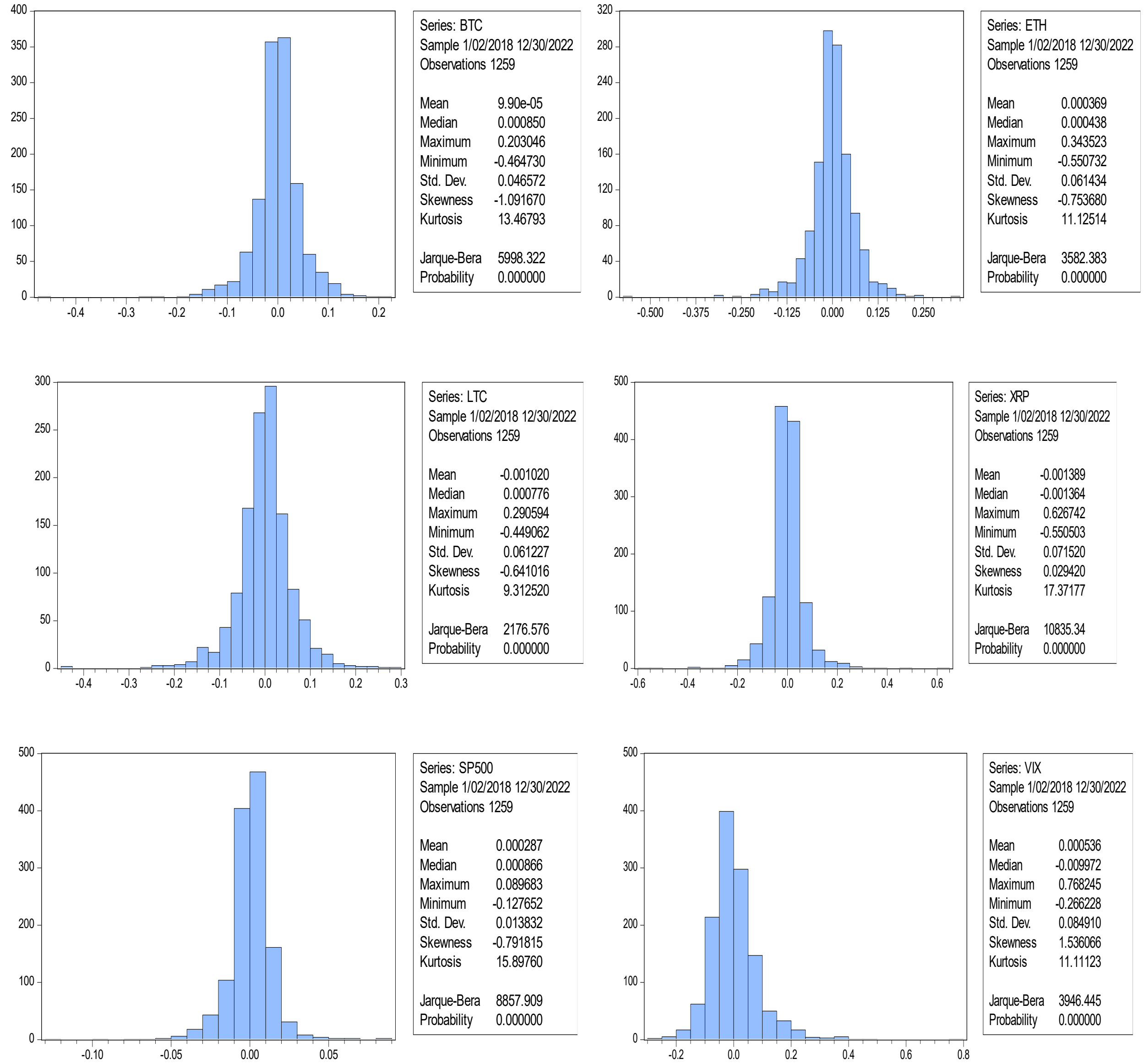

Figure 2. Histogram distribution of four cryptocurrencies, SP500, and VIX (2018 to 2022)

Figure 3 illustrates the volatility dynamics of the four cryptocurrencies, the S&P 500, and the VIX index following logarithmic transformation. The onset of the COVID-19 pandemic in early 2020 exerted a substantial impact on the volatility of all six variables. During 2021 and 2022, cryptocurrencies exhibit clear volatility clustering, whereby periods of high volatility tend to be followed by subsequent high-volatility episodes, while low-volatility periods persist over time. This persistence in volatility, as depicted in Figure 3, reflects the presence of time-dependent heteroskedasticity and suggests that volatility shocks have prolonged effects, potentially influencing market behavior and price dynamics.

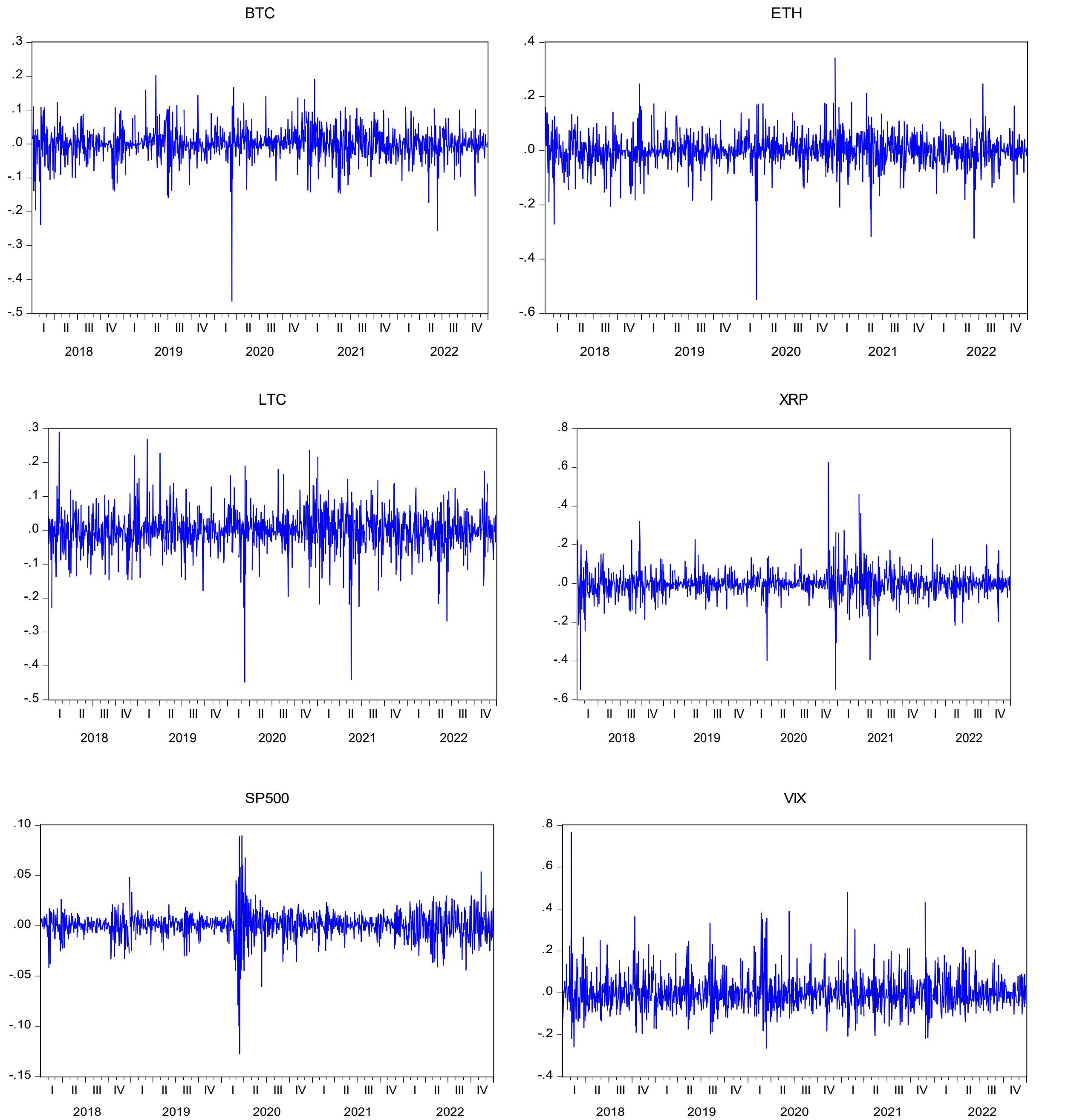

Figure 3. Daily return volatility of four cryptocurrencies, SP500, and VIX (2018 to 2022)

4.2 Correlation analysis

The untabulated correlation matrix reveals a strong and positive association among the four cryptocurrencies, indicating that they tend to move together. Among them, Ethereum (ETH) and Bitcoin (BTC) exhibit the highest correlation coefficient at 0.8233, followed by BTC and Litecoin (LTC) at 0.8117, suggesting strong synchronous price movements within the cryptocurrency market. Correlations between the four cryptocurrencies and the S&P 500 index are positive and statistically significant but relatively moderate, falling within the range of 0.2 to 0.3, consistent with partial co-movement between cryptocurrency and equity markets.

In contrast, the VIX index displays strong and negative correlations with all other variables, most notably with the S&P 500 index (correlation = −0.7151). This inverse relationship suggests that increases in the VIX—commonly interpreted as heightened market fear or uncertainty—are associated with declines in equity prices. The negative correlations observed between the VIX and the four cryptocurrencies imply that elevated market stress or negative investor sentiment tends to coincide with downward pressure on cryptocurrency prices as well. These patterns are consistent with risk-off episodes during which investors simultaneously withdraw from both traditional and digital asset markets.

4.3 Unit Root Tests and Cointegration Analysis

Using the augmented Dickey–Fuller (ADF) test, the untabulated results indicate that all six variables are stationary in their return series and do not contain unit roots. These findings are further supported by the Phillips–Perron (PP) test, where the t-statistics for all variables exceed the 1% critical threshold, reinforcing the conclusion that the variables are stationary. Additionally, the Lagrange Multiplier (LM) test demonstrates that all p-values are below 1%, confirming the presence of ARCH effects in all six variables. This validates the appropriateness of modeling the volatility dynamics using GARCH-type models.

Furthermore, unit root tests confirm that all six series are integrated of order one, I(1), satisfying the prerequisite for conducting cointegration tests. Table 3 presents the Johansen cointegration test results. Both the trace statistic and the maximum eigenvalue statistic reject the null hypothesis of no cointegration at the 1% significance level. These results suggest the existence of at least one long-run equilibrium relationship among the variables examined.

When comparing the test statistics against the 1% critical values, the results indicate that, at most, two cointegration vectors are rejected across the different combinations of the four cryptocurrencies, the S&P 500 index, and the VIX index. This provides strong evidence of cointegration among these variables, implying the presence of a stable long-term relationship linking the cryptocurrency market, the U.S. equity market, and investor sentiment. Establishing these long-run linkages provides a sound foundation for the subsequent volatility and spillover analyses using GARCH-class models.

Table 3. Cointegration test

Cointegration test of BTC / SP500 / VIX						
Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	1 Percent Critical Value	Hypothesized No. of CE(s)	Max-Eigen Statistic	1 Percent Critical Value
None ***	0.19868	742.131	35.65	None ***	277.747	25.52
At most 1 ***	0.18246	464.383	20.04	At most 1 ***	252.618	18.63
At most 2 ***	0.15538	211.765	6.65	At most 2 ***	211.765	6.65
Cointegration test of ETH / SP500 / VIX						
Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	1 Percent Critical Value	Hypothesized No. of CE(s)	Max-Eigen Statistic	1 Percent Critical Value
None ***	0.20509	754.953	35.65	None ***	287.8196	25.52
At most 1 ***	0.18524	467.134	20.04	At most 1 ***	256.9034	18.63
At most 2 ***	0.15435	210.230	6.65	At most 2 ***	210.2304	6.65
Cointegration test of LTC / SP500 / VIX						
Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	1 Percent Critical Value	Hypothesized No. of CE(s)	Max-Eigen Statistic	1 Percent Critical Value
None ***	0.20038	754.206	35.46	None ***	280.409	25.86
At most 1 ***	0.18348	473.796	19.94	At most 1 ***	254.189	18.52
At most 2 ***	0.16065	219.607	6.63	At most 2 ***	219.607	6.63
Cointegration test of XRP / SP500 / VIX						
Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	1 Percent Critical Value	Hypothesized No. of CE(s)	Max-Eigen Statistic	1 Percent Critical Value
None ***	0.19922	715.813	35.65	None ***	278.592	25.52
At most 1 ***	0.17021	437.221	20.04	At most 1 ***	233.966	18.63
At most 2 ***	0.14963	203.254	6.65	At most 2 ***	203.254	6.65

Note: The Trace Test and Max-Eigenvalue Test indicate the presence of 3 cointegrating equations at the 1% level (***), denoting rejection of the hypothesis at the 1% significance level.

4.4 Regression Analysis

4.4.1 Vector Autoregression (VAR)

After transforming all variables into first-order differences, a vector autoregression (VAR) model is estimated using the six return series. The results presented in Table 4 indicate that the returns of all variables are significantly influenced by their own first- and second-lagged values, confirming the presence of autoregressive dynamics within each series. This supports the suitability of a multivariate VAR framework for modeling the joint behavior of the six financial variables.

Furthermore, the results show strong bidirectional interactions between the S&P 500 index and the VIX index. Specifically, the returns of the S&P 500 and the VIX exert statistically significant effects on each other's lagged returns, suggesting the presence of return spillover effects between U.S. equity market performance and investor sentiment. This finding is consistent with the established inverse relationship between the two variables and highlights the interconnected nature of market risk and stock market behavior.

Table 4. VAR regression results of all Variables

VAR regression results of all Variables (1st difference)

	D.BTC	D.ETH	D.LTC	D.XRP	D.SP500	D.VIX
D.BTC(-1)	-0.746***	-0.147**	-0.106	-0.060	-0.014	-0.010
	(0.000)	(0.023)	(0.105)	(0.428)	(0.319)	(0.912)
D.BTC(-2)	-0.383***	-0.117*	-0.132**	-0.065	0.001	-0.050
	(0.000)	(0.070)	(0.043)	(0.395)	(0.958)	(0.589)
D.ETH(-1)	-0.018	-0.666***	-0.057	-0.036	0.004	-0.077
	(0.663)	(0.000)	(0.295)	(0.563)	(0.722)	(0.309)
D.ETH(-2)	0.045	-0.302***	0.047	-0.025	0.007	0.003
	(0.269)	(0.000)	(0.379)	(0.695)	(0.546)	(0.964)
D.LTC(-1)	0.016	0.002	-0.615***	-0.076	-0.009	0.140*
	(0.690)	(0.975)	(0.000)	(0.221)	(0.456)	(0.062)
D.LTC(-2)	-0.045	0.027	-0.306***	0.111*	-0.008	0.083
	(0.268)	(0.612)	(0.000)	(0.075)	(0.515)	(0.267)
D.XRP(-1)	0.021	0.032	0.013	-0.573***	0.008	-0.045
	(0.381)	(0.311)	(0.674)	(0.000)	(0.242)	(0.316)
D.XRP(-2)	0.023	-0.006	-0.013	-0.347***	0.006	-0.090**
	(0.325)	(0.851)	(0.678)	(0.000)	(0.371)	(0.043)
D.SP500(-1)	-0.209	-0.329*	-0.129	-0.289	-0.922***	0.891***
	(0.113)	(0.058)	(0.462)	(0.157)	(0.000)	(0.000)
D.SP500(-2)	-0.030	-0.030	-0.103	-0.309	-0.360***	0.497**
	(0.819)	(0.861)	(0.555)	(0.130)	(0.000)	(0.044)
D.VIX(-1)	-0.022	-0.041	-0.003	-0.041	-0.020***	-0.624***
	(0.273)	(0.119)	(0.901)	(0.191)	(0.001)	(0.000)
D.VIX(-2)	-0.006	-0.023	-0.028	-0.074**	-0.000	-0.322***
	(0.777)	(0.394)	(0.304)	(0.019)	(0.945)	(0.000)
_cons	-0.000	-0.000	0.000	-0.000	0.000	0.000
	(0.987)	(0.963)	(0.986)	(0.930)	(0.997)	(0.993)
N	1256	1256	1256	1256	1256	1256

Note: In parentheses, (-1) represents a lag of one period, and (-2) represents a lag of two periods.
P-values: * p < 0.1, ** p < 0.05, *** p < 0.01.

4.4.2 Granger causality test

To further examine dynamic interactions among the variables, Granger causality tests are conducted between the four cryptocurrencies and the two financial indicators—the VIX index and the S&P 500 index. The untabulated results reveal several significant causal relationships:

- BTC → XRP, ETH → XRP, and LTC → XRP, indicating that Bitcoin, Ethereum, and Litecoin all Granger-cause the returns of Ripple.

- BTC → S&P 500 and ETH → S&P 500, showing that Bitcoin and Ethereum exert predictive power over U.S. equity market returns.

- XRP → VIX, suggesting that Ripple contains information relevant to future changes in investor sentiment.

- S&P 500 ↔ VIX, indicating strong bidirectional causality between equity market returns and volatility expectations.

These results provide empirical support for the first two research questions, confirming that both return spillovers and directional causal relationships exist among the six variables.

An additional analysis using the first-order differenced series reveals further bidirectional causal relationships. In particular, the differenced Bitcoin series (DBTC) and the differenced XRP series (DXRP) exhibit mutual Granger causality, denoted as DBTC ↔ DXRP. Similarly, the differenced Litecoin series (DLTC) and DXRP display bidirectional causality (DLTC ↔ DXRP). These findings reinforce the presence of strong dynamic linkages and co-movement patterns among major cryptocurrencies.

4.4.3 EGARCH model

We apply the EGARCH(1,1) model to examine volatility asymmetry in the returns of four cryptocurrencies concerning SP500 and VIX. We initially focus on asymmetry effects within the four cryptocurrencies, assessing whether they exhibit volatility asymmetry in their returns. In Table 10's variance equation results, we use α_{ji}=C4 for assessing the shock effect of cryptocurrency i at period j and β_{ki} =C6 to evaluate the volatility effect of cryptocurrency i at period k, respectively. γ_{ji}=C5 indicates the presence of asymmetric effects in returns. Table 5 indicates that C4 and C6 are significant and different from zero, suggesting that all four cryptocurrencies are influenced by their prior unexpected shocks and volatility. Additionally, the coefficient C5 for the four cryptocurrencies is significantly negative, implying that each cryptocurrency displays distinctive asymmetric effects in returns.

Table 5. EGARCH results for four cryptocurrencies with SP500 and VIX

Phase 1 ~ Phase 4

	BTC				ETH			
	Mean Equation =C(1) + C(2)* Crypto(-1)							
	Phase1	Phase2	Phase3	Phase4	Phase1	Phase2	Phase3	Phase4
C(1)	-0.001	-0.001	0.002	-0.003	0.001	-0.003	0.007	-0.003
C(2)	0.016	0.026	-0.057	-0.076***	0.019	0.040	-0.057	0.033***
	Variance Equation GARCH = LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(5) *RESID(-1)/@SQRT(GARCH(-1)) + C(6)*LOG(GARCH(-1))							
	Phase1	Phase2	Phase3	Phase4	Phase1	Phase2	Phase3	Phase4
C(3)	-1.115	-1.100	-0.866	-4.840	-0.882	-0.275	-0.829	-1.976
C(4)	0.271***	0.260***	0.216***	0.304***	0.231***	0.078***	0.267***	0.431***
C(5)	-0.092***	-0.021	-0.109***	-0.279***	-0.060***	-0.026	-0.082***	-0.290***
C(6)	0.850***	0.853***	0.882***	0.280***	0.873***	0.962***	0.884***	0.715***
Loglikelihood	2126.896	894.969	856.057	386.773	1774.495	760.038	702.408	323.985
Durbin Watson	2.113	2.094	2.019	1.855	2.122	2.065	2.092	2.010

	LTC				XRP			
	Mean Equation =C(1) + C(2)* Crypto(-1)							
	Phase1	Phase2	Phase3	Phase4	Phase1	Phase2	Phase3	Phase4
C(1)	-0.002	-0.003	0.002	-0.002	-0.004	-0.003	-0.004	-0.005
C(2)	0.020	0.059	-0.044	-0.129***	-0.021	-0.011	-0.007	-0.024
	Variance Equation GARCH = LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(5) *RESID(-1)/@SQRT(GARCH(-1)) + C(6)*LOG(GARCH(-1))							
	Phase1	Phase2	Phase3	Phase4	Phase1	Phase2	Phase3	Phase4
C(3)	-1.106	-0.307	-0.730	-1.837	-0.627	-0.324	-0.633	-2.593
C(4)	0.226***	0.030	0.264***	0.423***	0.334***	0.139***	0.364***	0.576***
C(5)	-0.055***	-0.034	-0.026	-0.131***	0.060***	0.012	0.094***	-0.393***
C(6)	0.832***	0.950***	0.900***	0.741***	0.927***	0.961***	0.923***	0.646***
Log likelihood	1764.2	749.1	698.5	332.1	1690.2	751.3	613.0	353.1
DurbinWatson	2.122	2.000	2.097	1.944	1.964	1.952	1.979	2.124

	SP500				VIX			
	Mean Equation =C(1) + C(2)* Crypto(-1)							
	Phase1	Phase2	Phase3	Phase4	Phase1	Phase2	Phase3	Phase4
C(1)	0.001	0.001	0.001	0.001	0.004	0.005	0.004	-0.001
C(2)	-0.050*	-0.020	-0.097***	-0.050***	-0.065**	-0.035	-0.093**	-0.018
	Variance Equation GARCH = LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(5) *RESID(-1)/@SQRT(GARCH(-1)) + C(6)*LOG(GARCH(-1))							
	Phase1	Phase2	Phase3	Phase4	Phase1	Phase2	Phase3	Phase4
C(3)	-0.779	-0.860	-1.021	-0.772	-0.549	-0.213	-0.588	-0.802
C(4)	0.324***	0.213***	0.409***	0.172***	0.062***	-0.065***	0.018	0.260*
C(5)	-0.149***	-0.241***	-0.118***	-0.110***	0.253***	0.274***	0.248***	0.092
C(6)	0.942***	0.928***	0.921***	0.937***	0.901***	0.947***	0.884***	0.894***
Log likelihood	3970.3	1790.9	1605.0	706.2	1439.4	593.3	564.0	297.2
Durbin Watson	2.253	2.020	2.361	2.012	2.059	2.061	2.059	2.063

Note:

P value: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Phase 1 indicates full samples: January 1, 2018, to December 31, 2022. Phase 2 - Early COVID-19 Period: January 1, 2018, to January 29, 2020. Phase 3 - During the COVID-19 Pandemic: January 30, 2020, to February 23, 2022. Phase 4 - Post-COVID-19 Pandemic: February 24, 2022, to December 31, 2022.

In Table 6, φ_{1i} and φ_{2i} are used to examine whether there is a volatility spillover effect from the SP500 index and the VIX index on the conditional variance of the i-th cryptocurrency, respectively. In the variance equations of both Panel A and Panel B, α_{ji}(C5) examines the shock effect for the i-th cryptocurrency at time j, β_{ki}(C7) evaluates the volatility effect for the i-th cryptocurrency at time k, γ_{ji}(C6) represents the presence of asymmetric effects. φ_{1i} (SP500) or φ_{2i}(VIX) correspond to C8. In Panels A and B of Table 6, during the entire sample period, most C6 coefficients are non-significant, suggesting that past SP500 index volatility doesn't significantly impact the current asymmetric spillover effects of BTC, ETH, LTC, and VIX on ETH and LTC across this period. Additionally, the C8 coefficients for past SP500 index volatility concerning the returns of the four cryptocurrencies are all significantly negative. This indicates that they generate reverse (negative) volatility spillover effects.

In the variance equations of Panel C in Table 6, α_{ji}(C6) is employed to analyze the shock effect for the i-th cryptocurrency at time j, $\beta_{k,i}$(C8) evaluates the volatility effect for the i-th cryptocurrency at time k, γ_{ji}(C7) represents the presence of asymmetric effects. φ_{1i} and φ_{2i} correspond to C9 and C10, respectively. From Phase 3 (Covid-19 pandemic) to Phase 4 (post-Covid-19), the p-values of the C6 coefficients show a significant relationship. This suggests significant volatility asymmetry in the returns of the four cryptocurrencies concerning SP500 and VIX during this specific timeframe, indicating the presence of asymmetric effects. Moreover, During the Covid-19 pandemic (Phase 3), positive C7 coefficients for SP500 and VIX on the returns of the four cryptocurrencies suggest that market volatility has a stronger impact when there is positive news compared to negative news. In essence, positive news during the pandemic leads to a larger spillover effect of volatility than negative news. In the post-Covid-19 period, negative C7 coefficients for SP500 and VIX on the returns of the four cryptocurrencies imply that market volatility has a stronger impact when there is negative news compared to positive news. In other words, negative news about the U.S. stock market has a more significant effect on cryptocurrencies than positive news does.

When analyzing the asymmetry effect in the EGARCH model (Table 7), most γ coefficients are negative, and α-γ (C4-C5) is larger than α+γ (C4+C5). This signifies asymmetry in the returns of the four cryptocurrencies and suggests that the market's volatility impact from negative news is more substantial than the impact from positive news. In addition, the α-γ values (C4-C5) are higher during the post-Covid-19 period compared to other periods, highlighting the increased influence of negative news on volatility and spillover effects during this time. Notably, XRP has the highest value at 0.969, indicating a significant impact of negative news on XRP's conditional variance during this period. This finding aligns with the heightened cryptocurrency market volatility observed in the latter half of 2022, attributed to the FTX event, which led to a decline in cryptocurrency prices.

Table 6. EGARCH results of four cryptocurrencies with SP500 and VIX Pairing

Panel A: Four cryptocurrencies and SP500

Phase 1: Full samples

Panel A	BTC - SP500		ETH - SP500		LTC - SP500		XRP - SP500	
Mean Equation =C(1) + C(2)* Crypto(-1) +C(3)*SP500(-1)								
Variable	Coefficient	Prob.	Coefficient	Prob.	Coefficient	Prob.	Coefficient	Prob.
C(1)	0.0001	0.935	0.0009	0.577	-0.0015	0.361	-0.0030	0.055
C(2)	0.013	0.687	0.013	0.701	0.011	0.736	-0.044	0.177
C(3)	-0.171	0.105	-0.021	0.892	0.030	0.836	-0.053	0.680
Variance Equation LOG(GARCH) = C(4) + C(5)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(6) *RESID(-1)/@SQRT(GARCH(-1)) + C(7)*LOG(GARCH(-1)) + C(8) *SP500(-1)								
C(4)	-0.854***	0.000	-0.794***	0.000	-1.142***	0.000	-0.561***	0.000
C(5)	0.176***	0.000	0.174***	0.000	0.196***	0.000	0.290***	0.000
C(6)	0.001	0.926	0.021	0.246	0.004	0.871	0.107***	0.000
C(7)	0.883***	0.000	0.882***	0.000	0.822***	0.000	0.934***	0.000
C(8)	-11.625***	0.000	-12.057***	0.000	-10.212***	0.000	-11.171***	0.000
Log likelihood	2151.7		1797.2		1775.8		1712.5	
Durbin-Watson	2.076		2.106		2.108		1.913	

Panel B: Four cryptocurrencies and VIX

Phase 1: Full samples

Panel B	BTC - VIX		ETH - VIX		LTC - VIX		XRP - VIX	
Mean Equation =C(1) + C(2)* Crypto(-1) +C(3)*VIX(-1)								
Variable	Coefficient	Prob.	Coefficient	Prob.	Coefficient	Prob.	Coefficient	Prob.
C(1)	0.0002	0.899	0.0005	0.771	-0.0014	0.406	-0.0033	0.042
C(2)	-0.006	0.834	0.012	0.711	0.006	0.838	-0.043	0.188
C(3)	-0.005	0.793	-0.006	0.785	-0.013	0.557	-0.005	0.733
Variance Equation GARCH = LOG(GARCH) = C(4) + C(5)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(6)*RESID(-1)/@SQRT(GARCH(-1)) + C(7)*LOG(GARCH(-1)) + C(8)*VIX(-1)								
C(4)	-0.249***	0.000	-0.452***	0.000	-0.694***	0.000	-0.558***	0.000
C(5)	0.051***	0.000	0.130***	0.000	0.150***	0.000	0.283***	0.000
C(6)	0.015**	0.017	0.006	0.627	-0.002	0.903	0.104***	0.000
C(7)	0.966***	0.000	0.937***	0.000	0.896***	0.000	0.934***	0.000
C(8)	1.476***	0.000	1.458***	0.000	1.232***	0.000	1.524***	0.000
Log likelihood	2142.1		1788.0		1770.4		1703.6	
Durbin-Watson	2.072		2.112		2.105		1.923	

Panel C: Relationships of four cryptocurrencies with SP500 and VIX

Phase 1 ~ Phase 4

Panel C	BTC_SP500_VIX				ETH _SP500_VIX			
Mean Equation =C(1) + C(2)* Crypto(-1) +C(3)*SP500(-1) + C(4)*VIX(-1)								
	Phase1	Phase2	Phase3	Phase4	Phase1	Phase2	Phase3	Phase4
C(1)	0.0003	-0.0014	0.0030	-0.0016	0.0008	-0.0029	0.0072	-0.0048
C(2)	0.018	0.024	-0.039	-0.106*	0.003	0.039	-0.055	-0.001
C(3)	-0.125	-0.195	-0.332	-0.246	-0.096	-0.025	-0.227	-0.152
C(4)	-0.024	-0.041	-0.032	0.024	-0.020	-0.032	-0.031	0.050
Variance Equation LOG(GARCH) = C(5) + C(6)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(7)*RESID(-1)/@SQRT(GARCH(-1)) + C(8)*LOG(GARCH(-1)) + C(9) *SP500(-1) + C(10)*VIX(-1)								
	Phase1	Phase2	Phase3	Phase4	Phase1	Phase2	Phase3	Phase4
C(5)	-1.0237	-1.1604	-0.4487	-10.4994	-0.8684	-0.9384	-0.2756	-4.9559
C(6)	0.205***	0.256***	0.092***	0.508***	0.181***	0.127***	0.093***	0.580***
C(7)	-0.010	-0.019	0.046**	-0.314***	0.019	-0.002	0.114***	-0.522***
C(8)	0.858***	0.843***	0.937***	-0.532***	0.869***	0.852***	0.962***	0.241*
C(9)	-14.38***	-7.499	-10.49***	-4.554	-13.67***	-13.41***	-8.81***	5.800
C(10)	-0.612*	-0.208	0.541	-7.325***	-0.289	-0.067	0.704	-8.992***
Log likelihood	2153.1	897.3	881.5	400.7	1797.6	765.8	726.5	336.6
Durbin-Watson	2.069	2.098	2.020	1.674	2.090	2.080	2.079	1.836

Panel C	LTC_SP500_VIX				XRP _SP500_VIX			
Mean Equation =C(1) + C(2)* Crypto(-1) +C(3)*SP500(-1) + C(4)*VIX(-1)								
	Phase1	Phase2	Phase3	Phase4	Phase1	Phase2	Phase3	Phase4
C(1)	-0.0014	-0.0034	0.0038	-0.0025	-0.0030	-0.0027	0.0017	-0.0055
C(2)	0.008	0.050	-0.062	-0.220	-0.044	-0.014	-0.010	-0.073
C(3)	-0.044	0.195	-0.254***	-0.186	-0.061	-0.360	-0.271***	0.012
C(4)	-0.019	0.021	-0.039	-0.036	-0.002	-0.028	0.001	-0.045
Variance Equation LOG(GARCH) = C(5) + C(6)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(7)*RESID(-1)/@SQRT(GARCH(-1)) + C(8)*LOG(GARCH(-1)) + C(9) *SP500(-1) + C(10)*VIX(-1)								
	Phase1	Phase2	Phase3	Phase4	Phase1	Phase2	Phase3	Phase4
C(5)	-1.2745	-0.0532	-0.2194	-3.5179	-0.0030	-0.3337	-0.2776	-1.7239
C(6)	0.210***	-0.055***	0.078***	0.215**	-0.044	0.128***	0.199***	0.342***
C(7)	-0.002	-0.005	0.122***	-0.271***	-0.061	0.021	0.204***	-0.259***
C(8)	0.800***	0.984***	0.970***	0.446***	-0.002	0.958***	0.971***	0.766***
C(9)	-12.1***	-2.834*	-1.863	-42.9***	-11.22***	-0.220	-13.58***	-27.37***
C(10)	-0.371	0.920***	1.401**	-13.09***	-0.007	0.891*	-0.347	-5.440**
Log likelihood	1776.3	750.0	717.8	341.5	1712.5	754.5	639.4	359.4
Durbin-Watson	2.107	2.001	2.051	1.769	1.913	1.950	1.949	2.087

Note :

P value: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Phase 1 indicates full samples: January 1, 2018, to December 31, 2022. Phase 2 - Early COVID-19 Period: January 1, 2018, to January 29, 2020. Phase 3 - During the COVID-19 Pandemic: January 30, 2020, to February 23, 2022. Phase 4 - Post-COVID-19 Pandemic: February 24, 2022, to December 31, 2022.

Table 7. Summary of EGARCH empirical results

	BTC				ETH			
	Phase1	Phase2	Phase3	Phase4	Phase1	Phase2	Phase3	Phase4
C(4)=α	0.271	0.260	0.216	0.304	0.231	0.078	0.267	0.431
C(5)= γ	-0.092	-0.021	-0.109	-0.279	-0.060	-0.026	-0.082	-0.290
α+γ	0.180	0.240	0.106	0.025	0.172	0.052	0.185	0.141
α-γ	0.363	0.281	0.325	0.584	0.291	0.103	0.349	0.721
	LTC				XRP			
	Phase1	Phase2	Phase3	Phase4	Phase1	Phase2	Phase3	Phase4
C(4)=α	0.226	0.030	0.264	0.423	0.334	0.139	0.364	0.576
C(5)= γ	-0.055	-0.034	-0.026	-0.131	0.060	0.012	0.094	-0.393
α+γ	0.171	-0.004	0.238	0.292	0.394	0.152	0.458	0.183
α-γ	0.281	0.063	0.289	0.554	0.274	0.127	0.269	0.969

4.5 Additional Analysis

4.5.1 BEKK-GARCH model

The BEKK-GARCH estimation results reported in Table 8 (Panels A and B) indicate that volatility spillover effects among the four cryptocurrencies are not statistically significant in the mean equation. However, the diagonal elements of the conditional variance matrix—specifically A(1,1) and A(2,2)—are statistically significant across all models. This suggests that the volatility of each cryptocurrency is driven primarily by its own short-term shocks (ARCH effects) and long-term volatility persistence (GARCH effects). These effects are particularly pronounced for the S&P 500 and the VIX index, both of which exhibit strong and significant ARCH and GARCH coefficients.

Regarding the conditional variance dynamics, the diagonal parameters A(1,1) and A(2,2) (self-ARCH effects) and B(1,1) and B(2,2) (self-GARCH effects) are consistently significant, confirming the presence of strong own-market volatility dependence. When examining the interactions between cryptocurrency pairs and their linkages with the S&P 500 and VIX, all combinations exhibit significant influences from their own recent shocks and volatility persistence. Notably, the ARCH and GARCH parameters for both S&P 500 and VIX are highly significant, indicating substantial internal and cross-market volatility effects.

Moreover, the off-diagonal elements—A(1,2), A(2,1), B(1,2), and B(2,1)—are generally significant, reflecting meaningful cross-market volatility transmission among cryptocurrency pairs and between cryptocurrencies and the S&P 500 or VIX. Among the six cryptocurrency pairs, only the BTC–ETH pair shows no significant cross-market volatility interaction. In contrast, the BTC–LTC, ETH–LTC, and ETH–XRP pairs exhibit bidirectional volatility transmission, while the remaining pairs experience unidirectional spillovers. These findings collectively indicate strong cross-market volatility interdependence among the cryptocurrency pairs.

Panel C of Table 8 and the results in Table 9 further show that the S&P 500 index exerts a negative unidirectional spillover effect—in both short-term shocks and long-term volatility persistence—on the returns of all cryptocurrencies, with the exception of the XRP–S&P 500 pair. Similarly, the VIX index exhibits substantial spillover effects through both short-term shocks and long-term volatility on all four cryptocurrencies. In the short run, the spillover effects from VIX are primarily negative and unidirectional, with the exception of the BTC–VIX pair. Regarding long-term volatility transmission, the VIX displays bidirectional spillovers with ETH and LTC, and unidirectional spillovers with XRP. There are also significant bidirectional volatility spillover effects between the VIX and the S&P 500, highlighting strong interdependence between equity market uncertainty and overall market volatility.

Overall, the multivariate BEKK-GARCH model reveals that all series exhibit significant spillover effects from their own short-term shocks (ARCH) and long-term persistent volatility (GARCH). Among the cryptocurrency pairs, BTC–LTC, ETH–LTC, and ETH–XRP show bidirectional volatility transmission, while the remaining pairs display unidirectional effects. The S&P 500 index demonstrates a negative unidirectional volatility spillover on all four cryptocurrencies, whereas the VIX index exerts both unidirectional and bidirectional volatility spillovers through short-term and long-term mechanisms. These findings underscore the complex and interconnected volatility dynamics linking cryptocurrency markets with traditional financial markets.

Table 8. BEKK- GARCH of volatility spillover between pairs

Panel A _ (BTC-ETH/BTC-LTC/BTC-XRP – pairs) / Phase 1 Full samples

Variable	BTC - ETH		BTC - LTC		BTC - XRP	
			Variance Equation			
C(1,1)	0.0140***	0.000	0.0232***	0.000	0.0114***	0.000
C(2,1)	0.0151***	0.000	0.0447***	0.000	0.0103***	0.000
C(2,2)	0.0011	0.837	0.0149***	0.000	0.0104***	0.000
A(1,1)	0.4084***	0.000	0.0484	0.572	0.2315***	0.000
A(1,2)	-0.0722	0.244	-0.6963***	0.000	-0.3229***	0.000
A(2,1)	-0.0378	0.309	0.1900***	0.000	-0.0173	0.444
A(2,2)	0.2762***	0.000	0.5987***	0.000	0.5462***	0.000
B(1,1)	0.8715***	0.000	1.1411***	0.000	0.9197***	0.000
B(1,2)	-0.0187	0.429	0.5796***	0.000	0.0233	0.419
B(2,1)	0.0188	0.167	-0.3553***	0.003	0.0263***	0.001
B(2,2)	0.9545***	0.000	0.1047	0.561	0.8962***	0.000
Log likelihood	4676.9		4589.5		3203.5	
N	1258		1258		1258	

Panel B _ (ETH_LTC/ETH-XRP/LTC-XRP /SP500-VIX– pairs) / Phase 01 Full samples

Variable	ETH - LTC		ETH - XRP		LTC - XRP		SP500 - VIX	
				Variance Equation				
C(1,1)	0.0095***	0.006	0.0304***	0.000	0.0267***	0.000	0.0021***	0.000
C(2,1)	0.0194***	0.000	0.0245***	0.000	0.0293***	0.000	-0.0289***	0.000
C(2,2)	0.0000	1.000	0.0000	1.000	0.0000	1.000	0.0053	0.267
A(1,1)	0.3730***	0.000	0.5281***	0.000	-0.0454	0.348	0.3352***	0.000
A(1,2)	0.0006	0.994	0.2192***	0.000	-0.4859***	0.000	0.2673	0.212
A(2,1)	-0.0964	0.190	-0.1574***	0.000	0.2326***	0.000	-0.0172***	0.000
A(2,2)	0.2575***	0.000	0.3124***	0.000	0.7262***	0.000	0.4080***	0.000
B(1,1)	0.7135***	0.000	0.7010***	0.000	0.9424***	0.000	0.9403***	0.000
B(1,2)	-0.2207***	0.000	-0.2540***	0.000	-0.0081	0.825	-0.2398**	0.019
B(2,1)	0.2686***	0.000	0.0900***	0.000	-0.0929***	0.000	0.0099***	0.001
B(2,2)	1.0874***	0.000	0.9708***	0.000	0.7934***	0.000	0.8333***	0.000
Log likelihood	4287.5		4019.9		3983.1		6080.9	
N	1258		1258		1258		1258	

Panel C _ (4 cryptocurrencies – SP500) / Phase 01 Full samples

Variable	BTC - SP500		ETH - SP500		LTC - SP500		XRP - SP500	
				Variance Equation				
C(1,1)	0.0107***	0.000	0.0125***	0.000	0.0135***	0.000	0.0160***	0.000
C(2,1)	0.0007	0.130	0.0004	0.594	0.0009	0.210	0.0005	0.230
C(2,2)	0.0022***	0.000	0.0024***	0.000	0.0022***	0.000	0.0022***	0.000
A(1,1)	0.2405***	0.000	0.2384***	0.000	0.1811***	0.000	0.3745***	0.000
A(1,2)	0.0015	0.822	0.0051	0.300	0.0006	0.897	0.0006	0.872
A(2,1)	0.5873***	0.000	0.6743***	0.000	0.5419***	0.000	0.2665**	0.035
A(2,2)	0.4825***	0.000	0.4827***	0.000	0.4910***	0.000	0.4556***	0.000
B(1,1)	0.9389***	0.000	0.9485***	0.000	0.9565***	0.000	0.9092***	0.000
B(1,2)	-0.0014	0.730	0.0001	0.969	-0.0022	0.571	-0.0010	0.609
B(2,1)	-0.2173***	0.000	-0.2603***	0.000	-0.1842***	0.000	-0.0727	0.195
B(2,2)	0.8710***	0.000	0.8650***	0.000	0.8658***	0.000	0.8849***	0.000
Log likelihood	6128.046		5784.349		5748.372		2143.239	
N	1258		1258		1258		1258	

Panel D _ (4 cryptocurrencies – VIX) / Phase 01 Full samples

Variable	BTC - VIX		ETH - VIX		LTC - VIX		XRP - VIX	
				Variance Equation				
C(1,1)	0.0133***	0.000	0.0226***	0.000	0.0262***	0.000	0.0053***	0.010
C(2,1)	-0.0172**	0.026	-0.0352***	0.000	-0.0203**	0.012	0.0398***	0.000
C(2,2)	0.0288***	0.004	0.0192***	0.000	0.0239***	0.007	0.0000	1.000
A(1,1)	0.3097***	0.000	0.2735***	0.000	0.2895***	0.000	0.4244***	0.000
A(1,2)	-0.0801	0.294	0.0092	0.864	-0.0198	0.674	0.0454	0.209
A(2,1)	0.0255	0.662	-0.1205***	0.000	0.0567**	0.089	0.0784***	0.002
A(2,2)	0.4167***	0.000	0.4834***	0.000	0.4225***	0.000	0.4652***	0.000
B(1,1)	0.8750***	0.000	0.8878***	0.000	0.8098***	0.000	0.8909***	0.000
B(1,2)	0.2642**	0.031	0.1049***	0.008	0.2909***	0.000	-0.0202	0.333
B(2,1)	-0.0718	0.159	0.1063***	0.000	-0.1151***	0.000	-0.1081***	0.000
B(2,2)	0.8379***	0.000	0.7642***	0.000	0.8501***	0.000	0.7622***	0.000
Log likelihood	3563.661		3232.936		3203.450		3112.000	
N	1258		1258		1258		1258	

Note: P value: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. In Panels A, B, C, and D, we do not list the results of Mean Equation.

Table 9. Volatility and Transmission Direction for Each Pairing in BEKK-GARCH

(4 cryptocurrencies – pairs, 4 cryptocurrencies – VIX or SP500) / hase 01 Full samples

	BTC-ETH	BTC-LTC	BTC-XRP	ETH-LTC	ETH-XRP	LTC-XRP
Mean Spillover	<——	——	——	——	<——	——
Shock Transmission	——	<——>	——>	——	<——>	<——>
Volatility Spillover	——	<——>	<——	<——>	<——>	<——
	BTC-SP500	**ETH-SP500**	**LTC-SP500**	**XRP-SP500**	**VIX-SP500**	
Mean Spillover	——	<——	——	——	<——	
Shock Transmission	<——	<——	<——	<——	——>	
Volatility Spillover	<——	<——	<——	——	<——>	
	BTC-VIX	**ETH-VIX**	**LTC-VIX**	**XRP-VIX**	**SP500-VIX**	
Mean Spillover	——	——	——	——	——>	
Shock Transmission	——	<——	<——	<——	<——	
Volatility Spillover	——>	<——>	<——>	<——	<——>	

4.5.2 DCC-GARCH model

The results of the DCC-GARCH model, presented in Table 10, indicate the presence of significant time-varying volatilities and dynamic correlations among the four cryptocurrencies, as well as between each cryptocurrency and the S&P 500 and VIX indices. For the full-sample period (Phase 01), the estimated dynamic correlation parameters α and β, along with the correlation coefficients ρ_(ij,t)(corr_cons), are statistically significant. In all cases, β exceeds α, suggesting that past volatility persistence contributes more to current conditional variance than do short-term shocks. Moreover, the sum of α and β is close to unity for all cryptocurrency pairs—most notably, the ETH–BTC pair, which exhibits the highest combined value of 0.997.

Similarly, when cryptocurrencies are paired with the S&P 500 or VIX, the combined α + β values remain close to one and are highly significant. This indicates a strong degree of persistence in dynamic correlations, implying that after a market disturbance, correlations do not quickly revert to their long-term unconditional levels. Instead, they remain elevated or depressed for extended periods, highlighting persistent interdependence across markets.

Table 10. Dynamic relationships in DCC-GARCH

Panel A

	(1) ETH - BTC	(2) LTC - BTC	(3) XRP - BTC	(4) LTC - ETH	(5) XRP - ETH	(6) XRP - LTC
Adjustment						
lambda1 (α)	0.074***	0.029***	0.051***	0.045***	0.053***	0.073***
lambda2 (ß)	0.9226***	0.924***	0.931***	0.915***	0.918***	0.900***
corr_cons	0.842***	0.841***	0.689***	0.862***	0.800***	0.807***
p-values	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
N	1258	1258	1258	1258	1258	1258

Panel B

	(1) (SP500-BTC)	(2) (SP500-ETH)	(3) (SP500-LTC)	(4) (SP500-XRP)
Adjustment				
lambda1 (α)	0.024***	0.036**	0.019***	0.043***
lambda2 (ß)	0.970***	0.953***	0.972***	0.910***
corr_cons	0.283**	0.251**	0.250***	0.211***
N	1258	1258	1258	1258

Panel C

	(1) VIX - BTC	(2) VIX - ETH	(3) VIX - LTC	(4) VIX - XRP
Adjustment				
lambda1 (α)	0.026***	0.031***	0.024***	0.025***
lambda2 (ß)	0.973***	0.961***	0.972***	0.964***
corr_cons	-0.779	-0.366***	-0.470*	-0.234***
N	1258	1258	1258	1258

Note: P value: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table 10 and Figure 4 illustrate the estimated dynamic correlations ρ_(ij,t)(corr_cons), among the four cryptocurrencies, as well as between each cryptocurrency and the S&P 500 or VIX. These correlations are statistically significant throughout the sample period. Among the cryptocurrency–VIX pairs, VIX–LTC exhibits only weak correlation, while VIX–BTC appears largely uncorrelated. By contrast, the dynamic correlations among the four cryptocurrencies are strongly positive, with correlation coefficients exceeding 0.800 for all pairs except the XRP–BTC pair, which still demonstrates a substantial correlation (0.689).

Figures 5 and 6 reveal several additional patterns. First, the S&P 500 displays a positive dynamic correlation with all four cryptocurrencies, suggesting increasing comovement between equity markets and digital assets. Second, the VIX exhibits a consistently negative dynamic correlation with cryptocurrencies, consistent with the interpretation of VIX as a fear index that rises during periods of heightened risk aversion.

Importantly, the dynamic correlations among the four cryptocurrencies remain relatively stable even after the onset of the COVID-19 pandemic. However, the correlation between the S&P 500 and cryptocurrencies strengthens over time, indicating increased integration between cryptocurrency and traditional equity markets. In contrast, the correlation between the VIX and cryptocurrencies becomes more negative during the pandemic period, suggesting that investors increasingly viewed cryptocurrencies as alternative assets during episodes of elevated uncertainty and market stress.

Figure 4. Dynamic correlation graph among four cryptocurrencies pairs

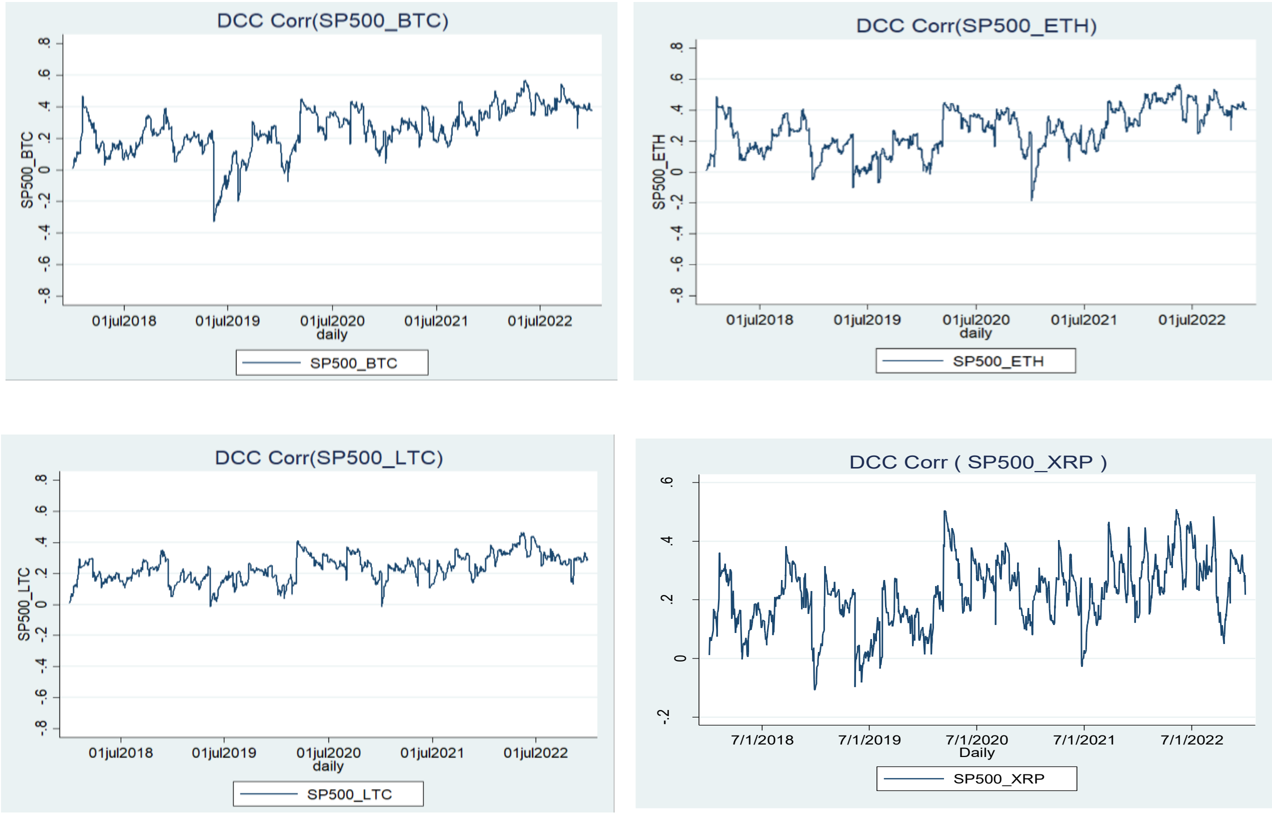

Figure 5. Dynamic correlation graph between four cryptocurrencies and SP500

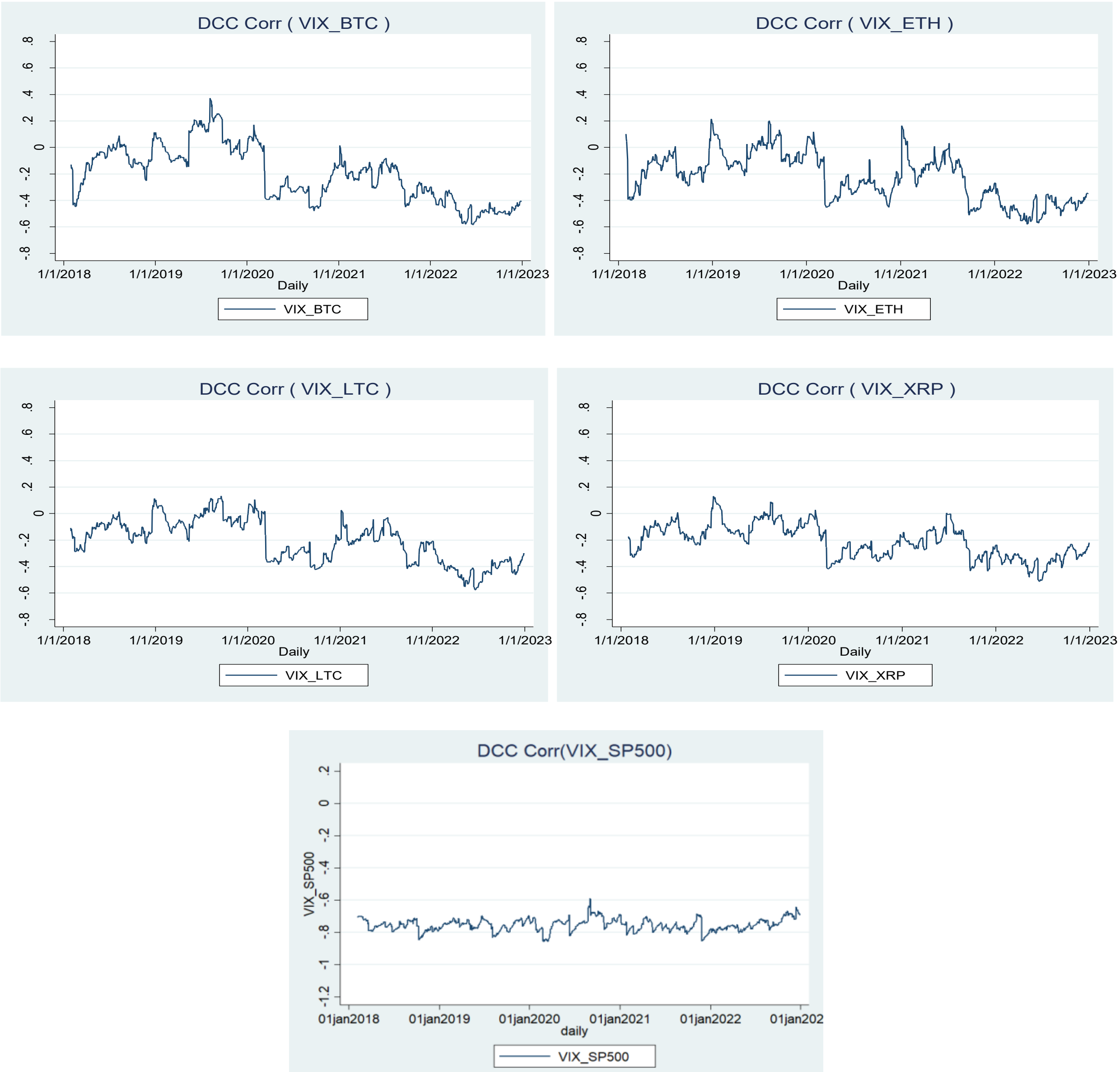

Figure 6. Dynamic correlation graph between four cryptocurrencies and VIX

4.5.3 Deep Learning LSTM Predictive Analysis

To complement the GARCH-family results, this study further employs a deep learning approach using a long short-term memory (LSTM) neural network to forecast future cryptocurrency prices. The dataset covers a total of 1,322 daily observations from January 1, 2018 to June 30, 2023. The first 1,199 observations (January 1, 2018 to December 31, 2022) are used for model training, while the remaining 123 observations (January 1, 2023 to June 30, 2023) serve as the test set.

The LSTM model is trained using a batch size of 32, 5 epochs, and a feature window of 60 days, implying that the network learns price patterns based on rolling 60-day segments of historical data. After training the model on the 1,199-day sample, the model is evaluated by forecasting the subsequent 123 days of testing data for all four cryptocurrencies. The resulting predictive performance is displayed in Figure 7.

When the LSTM model is augmented by incorporating additional features from the S&P 500 and VIX indices, the predicted values for the four cryptocurrencies more closely track actual movements, as shown in Figure 8. Notably, the inclusion of S&P 500 data results in an upward vertical shift in predicted values, whereas the inclusion of VIX data produces a downward shift. These shifts are consistent with earlier empirical findings—particularly the DCC-GARCH results—which reveal a positive dynamic correlation between the S&P 500 and the cryptocurrencies and a negative dynamic correlation between the VIX and cryptocurrencies.

Furthermore, the predictive patterns reinforce the earlier observation that dynamic correlations among the four cryptocurrencies remain relatively stable even after the COVID-19 pandemic. This suggests that, despite major macroeconomic shocks, the internal structure and interdependence of the cryptocurrency market exhibit a strong degree of persistence.

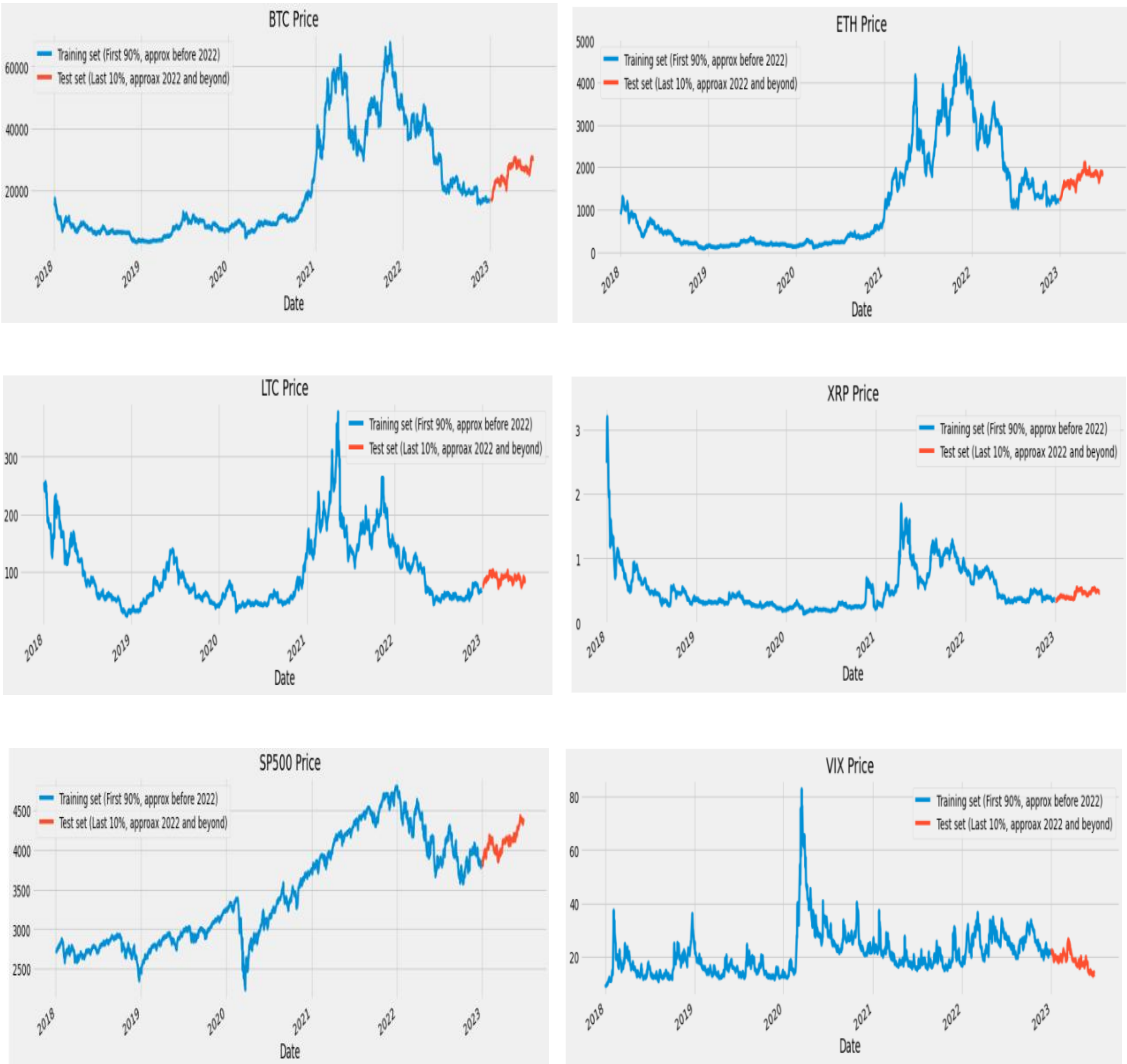

Figure 7. Predicting the future trends of cryptocurrencies, SP500, and VIX using LSTM

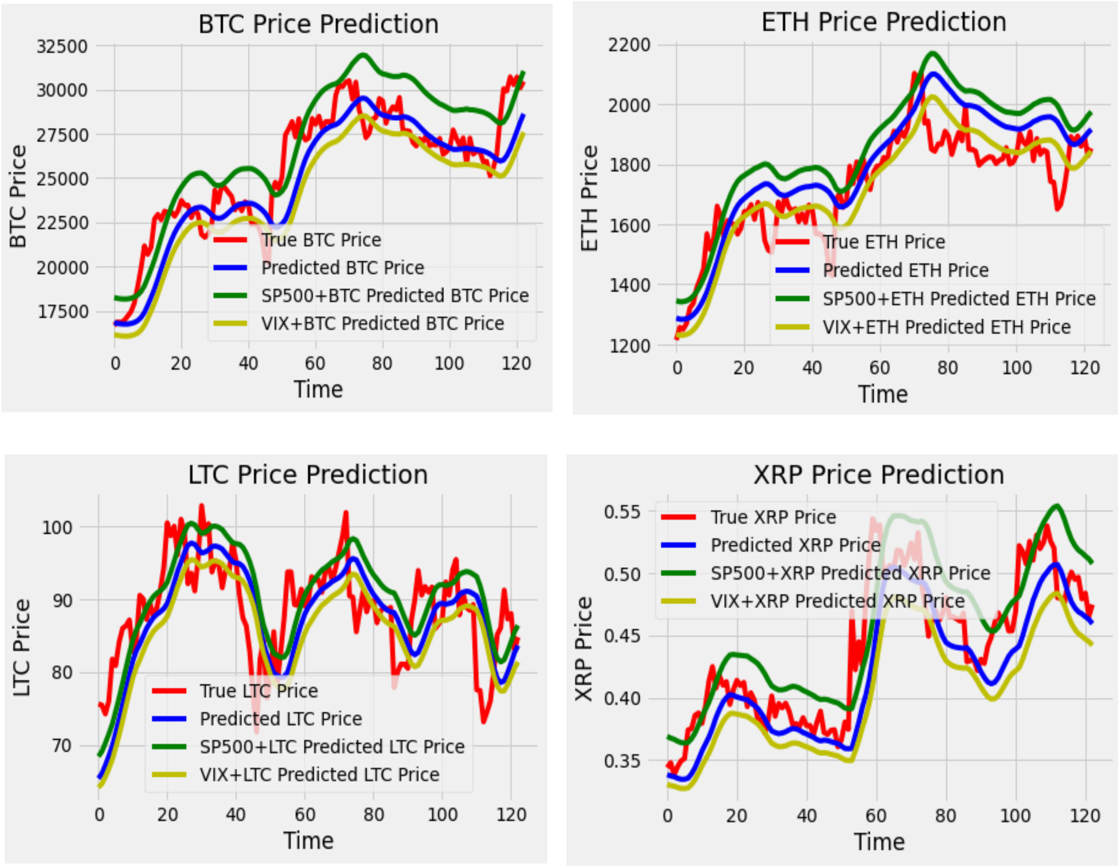

Figure 8. Predicting co-movement between cryptocurrencies, SP500, and VIX using LSTM

5. Conclusions

This study investigates the co-movement of returns and volatilities in cryptocurrency markets across three distinct phases: the pre-COVID-19 period, the COVID-19 pandemic, and the post-pandemic period. In addition, it examines volatility spillover effects among four major cryptocurrencies—Bitcoin, Ethereum, Litecoin, and Ripple—the S&P 500 index, and the VIX index. To achieve these objectives, a comprehensive empirical framework is employed, incorporating vector autoregression (VAR), univariate EGARCH, and multivariate BEKK and DCC-GARCH models, following a series of preliminary time-series property tests.

The empirical findings highlight the importance of volatility clustering in cryptocurrency portfolio construction. Elevated correlations in cryptocurrency volatility can substantially increase portfolio risk, particularly during periods of market stress. To mitigate this risk, investors may benefit from diversifying across different types of cryptocurrencies rather than concentrating on a single digital asset. Furthermore, the results demonstrate that the S&P 500 and VIX indices exert significant spillover effects on cryptocurrency market volatility. This underscores the necessity for investors to closely monitor global equity market conditions and investor sentiment indicators when managing cryptocurrency portfolios. Episodes of heightened stock market volatility or deteriorating market sentiment may transmit adverse effects to cryptocurrency markets. Consequently, diversification strategies that combine cryptocurrencies with traditional financial assets, such as equities and bonds, may help reduce the overall impact of volatility spillovers.

The study also identifies pronounced asymmetric market responses, whereby negative shocks exert a stronger influence on volatility than positive shocks. This asymmetry has important implications for risk management in cryptocurrency investments. Investors should account for this behavior when designing trading and hedging strategies, as risk exposure tends to increase disproportionately during market downturns. For instance, the use of stop-loss mechanisms may help limit losses during periods of heightened uncertainty and preserve portfolio stability.

Moreover, the four cryptocurrencies exhibit consistently high positive dynamic correlations, indicating strong interconnectedness in their price movements. The analysis further reveals dynamic volatility linkages between cryptocurrencies and the S&P 500 and VIX indices, suggesting that market volatility can propagate across asset classes in response to common economic, financial, or geopolitical shocks. These findings imply that cryptocurrency markets are increasingly integrated into the broader financial system and are influenced by similar macroeconomic forces.

Overall, the results indicate that cryptocurrency markets are highly sensitive to global economic conditions, shifts in investor sentiment, and major systemic events. Investors should incorporate considerations of volatility clustering, spillover effects, asymmetric responses, and cross-market correlations when making investment decisions. The empirical evidence presented in this study offers economically meaningful insights into cryptocurrency risk dynamics and contributes to a deeper understanding of market behavior, particularly during periods of heightened uncertainty such as the COVID-19 pandemic. These insights may assist investors in developing more effective risk management and portfolio allocation strategies.

Finally, this study is subject to certain limitations. The sample selection and time constraints, particularly the differences in trading hours across markets, may influence the results. While cryptocurrencies trade continuously, the S&P 500 and VIX are subject to limited trading days and hours, which may introduce measurement biases. Future research may address these limitations by employing high-frequency data, alternative sentiment indicators, or broader asset coverage to further explore cross-market dynamics and enhance the robustness of the findings.

References

Baur, D. G. and T. Dimpfl, (2018), “Asymmetric volatility in cryptocurrencies,” *Economics Letters*, 173, 148-151.

Bouri, E., M. Das, R. Gupta, and D. Roubaud, (2018), “Spillovers between Bitcoin and other assets during bear and bull markets,” *Applied Economics*, 50(55), 5935-5949.

Bouri, E., R. Gupta, and D. Roubaud, (2019), “Herding behaviour in cryptocurrencies,” *Finance Research Letters*, 29, 216-221.

Bouri, E., B. Lucey, and D. Roubaud, (2020), “Cryptocurrencies and the downside risk in equity investments,” *Finance Research Letters*, 33, 101211.

Conlon, T., S. Corbet, and R. J. McGee, (2020), “Are cryptocurrencies a safe haven for equity markets? An international perspective from the COVID-19 pandemic,” *Research in International Business and Finance*, 54, 101248.

da Gama Silva, P. V. J., M. C. Klotzle, A. C. F. Pinto, and L. L. Gomes, (2019), “Herding behavior and contagion in the cryptocurrency market,” *Journal of Behavioral and Experimental Finance*, 22, 41-50.

Dickey, D. A. and W. A. Fuller, (1979), “Distribution of the estimators for autoregressive time series with a unit root,” *Journal of the American statistical association*, 74(366a), 427-431.

Diebold, F. X. and K. Yilmaz, (2009), “Measuring financial asset return and volatility spillovers, with application to global equity markets,” *The Economic Journal*, 119(534), 158-171.

Diebold, F.X. and K. Yilmaz, (2012), “Better to give than to receive: predictive directional measurement of volatility spillover,” *International Journal of Forecasting*, 28, 57-66.

Dyhrberg, A. H., (2016), “Hedging capabilities of bitcoin. Is it the virtual gold?” *Finance research letters*, 16, 139-144.

Engle, R. F. and K. F. Kroner, (1995), “Multivariate simultaneous generalized ARCH,” *Econometric theory*, 11(1), 122-150.

Engle, R., (2002), “Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models,” *Journal of business & economic statistics*, 20(3), 339-350.

Ghorbel, A. and A. Jeribi, (2021), “Investigating the relationship between volatilities of cryptocurrencies and other financial assets,” *Decisions in Economics and Finance*, 44(2), 817-843.

Guesmi, K., S. Saadi, I. Abid, and Z. Ftiti, (2019), “Portfolio diversification with virtual currency: Evidence from bitcoin,” *International Review of Financial Analysis*, 63, 431-437.

Hsu, S. H., (2022), "Investigating the co-volatility spillover effects between cryptocurrencies and currencies at different natures of risk events," *Journal of Risk and Financial Management*, 15(9), 372.

Khan, M., U. N. Kayani, M. Khan, K. S. Mughal, and M. Haseeb, (2023), "COVID-19 pandemic & financial market volatility; evidence from GARCH models," *Journal of Risk and Financial Management*, 16(1), 50.

Koutmos, D., (2018), "Return and volatility spillovers among cryptocurrencies," *Economics Letters*, 173, 122-127.

Kumar, A. and T. Ajaz, (2019), "Co-movement in crypto-currency markets: evidences from wavelet analysis," *Financial Innovation*, 5(1), 1-17.

Kurka, J., (2019), "Do cryptocurrencies and traditional asset classes influence each other ?," *Finance Research Letters*, 31, 38-46.

Liu, J. and A. Serletis, (2019), "Volatility in the cryptocurrency market," *Open Economies Review*, 30(4), 779-811.

Özdemir, O., (2022), "Cue the volatility spillover in the cryptocurrency markets during the COVID-19 pandemic: evidence from DCC-GARCH and wavelet analysis," *Financial Innovation*, 8(1), 12.

Phillips, P. C. and P. Perron, (1988), "Testing for a unit root in time series regression," *biometrika*, 75(2), 335-346.

Qiao, X., H. Zhu, and L. Hau, (2020), "Time-frequency co-movement of cryptocurrency return and volatility: Evidence from wavelet coherence analysis," *International Review of Financial Analysis*, 71, 101541.

Song, J. Y., W. Chang, and J. W. Song, (2019), "Cluster analysis on the structure of the cryptocurrency market via Bitcoin–Ethereum filtering," *Physica A: Statistical Mechanics and its Applications*, 527, 121339.

Tiwari, A. K., I. D. Raheem, and S. H. Kang, (2019), "Time-varying dynamic conditional correlation between stock and cryptocurrency markets using the copula-ADCC-EGARCH model," *Physica A: Statistical Mechanics and Its Applications*, 535, 122295.

Urquhart, A. and H. Zhang, (2019), "Is Bitcoin a hedge or safe haven for currencies? An intraday analysis," *International Review of Financial Analysis*, 63, 49-57.

Zeng, T., M. Yang, and Y. Shen, (2020), "Fancy Bitcoin and conventional financial assets: Measuring market integration based on connectedness networks," *Economic Modelling*, 90, 209-220.